Power Transformer Modelling to Support the Interpretation of Frequency Response Analysis

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B.E. (Elec.)(Hons.)

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Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I give consent to this copy of my thesis, when deposited in the University Library, being made available for loan and photocopying subject to the provisions of the Copyright Act 1968.

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A_w	Cross	sectional	area	of	winding
					0

- A_{CL} Conductor cross sectional area lower bound
- A_{CSL} Lower bound for the core cross sectional area
- A_{CSU} Upper bound for the core cross sectional area
- A_{CS} Cross sectional area of the core
- A_{CU} Conductor cross sectional area upper bound
- *B* Magnetic flux density
- C_{SXi} Equivalent series winding capacitance for winding section *i* of phase X
- C_{XYi} Capacitance between the high voltage winding section i of phase X and the high voltage winding section i of phase Y
- C_{Xi} Capacitance for winding section *i* of phase X
- C_{Xxi} Capacitance between the high and low voltage windings of winding section *i* of phase X
- C_{gXi} Capacitance between the high voltage winding and the transformer tank for winding section i of phase X
- C_{gxi} Capacitance between the low voltage winding and the transformer core for winding section i of phase X
- D Electric flux density
- *D* High voltage delta connection
- E Electric field
- G Model transfer function
- G_{Xi} Conductance for winding section *i* of phase X

- *H* Frequency response
- H Magnetic field intensity
- I_{HR} Nameplate current for high voltage side
- I_{SCX} Short circuit current for the high voltage winding of phase X
- J Cost function
- J Current density
- K_V Transformer voltage gain
- K_i Transformer current gain
- L' Ferromagnetic inductance
- L_0 Inductance based on unity permeability
- L_{AA} Self inductance for a high voltage winding on an outside limb
- L_{BB} Self inductance for a high voltage winding on the centre limb
- L_{LA} Leakage inductance for the high voltage outside limb winding
- L_{LB} Leakage inductance for the high voltage centre limb winding
- L_{LX} High voltage winding leakage inductance of phase X
- $L_{LX}^{'}$ $\;$ Low voltage winding leakage inductance referred to the high voltage side of phase $\;$ X
- L_{La} Leakage inductance for the low voltage outside limb winding
- L_{Lb} Leakage inductance for the low voltage centre limb winding
- L_{Lx} Low voltage winding leakage inductance of phase X
- L_{MAA} Core component of the self inductance for the high voltage outside limb winding
- L_{MAB} Mutual inductance between a high voltage winding on the outside limb and a high voltage winding on the centre limb
- L_{MAC} Mutual inductance between the high voltage windings on opposing outside limbs
- ${\cal L}_{MAa}~$ Mutual inductance between the high and low voltage windings on the core's outside limb
- L_{MAb} Mutual inductance between a high voltage winding on the outside limb and a low voltage winding on the centre limb

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- L_{MAc} Mutual inductance between a high voltage winding on an outside limb and a low voltage winding on the opposing outside limb
- L_{MBB} Core component of the self inductance for the high voltage outside limb winding
- L_{MBb} Mutual inductance between the high and low voltage windings on the core's centre limb
- L_{Maa} Core component of the self inductance for the low voltage outside limb winding
- L_{Mab} Mutual inductance between a low voltage winding on the outside limb and a low voltage winding on the centre limb
- L_{Mac} Mutual inductance between the low voltage windings on opposing outside limbs
- L_{Mbb} Core component of the self inductance for the low voltage centre limb winding
- L_X Self inductance of winding X
- L_{aa} Self inductance for a low voltage winding on an outside limb
- L_{bb} Self inductance for a low voltage winding on the centre limb
- L_{mX} Magnetising inductance of phase X
- N_X Number of turns on the high voltage winding of phase X
- N_x Number of turns on the low voltage winding of phase X
- N_{DS} Number of discs per section
- N_{HL} Lower bound for the number of turns in the high voltage winding
- N_{HU} Upper bound for the number of turns in the high voltage winding
- N_{LL} Lower bound for the number of turns in the low voltage winding
- N_{LU} Upper bound for the number of turns in the low voltage winding
- N_{TD} Number of turns per disc
- Q Total free charge
- R'' Magnetic loss resistance

 R_{DCXi} DC winding resistance of winding section *i* of phase X

- R_{PXi} AC winding resistance due to proximity effect of winding section *i* of phase X
- R_{SXi} AC winding resistance due to skin effect of winding section *i* of phase X
- R_{TT} Measured resistance between two terminals

- R_T FRA termination resistor
- R_W Winding resistance
- R_X High voltage winding resistance of phase X
- R_x Low voltage winding resistance of phase X
- V Voltage
- Y Admittance
- Y High voltage star connection
- Z% Percent impedance
- Z_X HV winding impedance of phase X
- Z_x LV winding impedance of phase X
- Z_L Transformer impedance
- Δ Lamination stacking excess
- Δ_{EW} Mean orthogonal distance between the outside diameter of the high voltage winding and the transformer tank end wall
- Δ_{SW} Mean orthogonal distance between the outside diameter of the high voltage winding and the transformer tank side wall
- Δ_W Mean orthogonal distance between the outside diameter of the high voltage winding and the transformer tank walls
- Γ Inductive disparity ratio
- Λ Core dimension ratio
- Φ_L Leakage flux
- Φ_{MAB} Flux generated in an outside limb linking a winding on the centre limb
- Φ_{MAC} Flux generated in one outside limb linking a winding on the other outside limb
- Φ_{MA} Flux in the core's outside limb
- Φ_{MB} Flux in the core's centre limb
- Φ_X Flux observed by winding X
- Φ_m Magnetising flux
- Θ_{COD} Transformer core outside diameter

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- Θ_{HID} High voltage winding inside diameter
- Θ_{HL} Lower bound for the mean high voltage winding diameter
- Θ_{HOD} High voltage winding outside diameter
- Θ_{HU} Upper bound for the mean high voltage winding diameter
- Θ_{LID} Low voltage winding inside diameter
- \bar{L} Ferromagnetic base inductance for a winding section
- \bar{a} Transformer turns ratio
- δ Skin depth
- δ_{HV} Orthogonal distance between two adjacent high voltage windings
- δ_c Loss angle
- ϵ Electrical permittivity of the dielectric medium
- ϵ'' Imaginary part of complex permittivity
- ϵ' Real part of complex permittivity
- ϵ_c Complex permittivity
- γ Propagation constant
- \hat{A}_{CS} Estimate for the core cross sectional area
- \hat{A}_C Estimate for the conductor cross sectional area
- \hat{C} Geometric capacitance
- \hat{G} Estimated transfer function
- \hat{L}_{LXij} Mutual (leakage) inductance between winding sections *i* and *j* of phase X
- \hat{L}_{LXi} Leakage self inductance for winding sections *i* of phase X
- \hat{L}_{LX} Nominal high voltage leakage inductance of phase X
- \hat{L}_{LX} Nominal low voltage leakage inductance of phase X
- \hat{N}_H Estimate for the number of turns in the high voltage winding
- \hat{N}_L Estimate for the number of turns in the low voltage winding
- $\hat{\Theta}_H$ Estimate for the mean high voltage winding diameter
- $\hat{\theta}$ Estimation algorithm parameter solution set

- \hat{l}_C Estimate for the winding conductor length
- \hat{l}_E Estimate for the core limb length
- \hat{l}_Y Estimate for the core yoke length
- κ Winding connection scaling factor
- \mathcal{F}_X Magnetomotive force for the high voltage winding of phase X
- \mathcal{F}_m Resultant core magnetomotive force
- \mathcal{F}_x Magnetomotive force for the low voltage winding of phase X
- \mathcal{R}_E Transformer core limb reluctance
- \mathcal{R}_{FEX} Magnetic core leakage reluctance of phase X
- \mathcal{R}_L Winding leakage flux reluctance across the core window
- \mathcal{R}_{MA} Magnetic core reluctance for a winding on an outside limb
- \mathcal{R}_{MB} Magnetic core reluctance for a winding on the centre limb
- \mathcal{R}_{WX} Core window leakage reluctance of phase X
- \mathcal{R}_X Reluctance observed by the winding X
- \mathcal{R}_Y Transformer core yoke reluctance
- \mathcal{R}_m Core reluctance
- \mathfrak{C}_{Xi} Non ideal capacitance for winding section *i* of phase X
- \mathfrak{G}_{Xi} Composite conductance for winding section *i* of phase X
- \mathfrak{L}_{Xi} Composite inductance for winding section *i* of winding X
- \mathfrak{R}_{Xi} Composite series resistance element of winding section *i* of phase X
- μ_0 Permeability of free space
- μ_e Effective complex permeability of the core
- μ_i Initial permeability
- μ_r Relative permeability
- μ_s Effective complex relative permeability
- μ_s'' Imaginary component of μ_s
- μ'_s Real component of μ_s

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$\nabla \mathbf{x}$	Curl operator
$\phi(t)$	Net flux per inductance link fundamental loop
σ	Conductivity
au	Leakage coefficient of coupling
\tilde{L}	Generic inductance term
\triangle_{AV}	The average change in the interwinding capacitance difference relative to the base- line values.
b	Half of the lamination thickness
d	Conductor diameter
d	Low voltage delta connection
d_s	Distance between discs
e_m	Magnetising electromotive force
f	Frequency
i	Current
i_{HR}	Nameplate current rating for the high voltage winding
i_{LR}	Nameplate current rating for the low voltage winding
k	Lamination stacking factor
l_E	Transformer core mean limb length
l_Y	Transformer core mean yoke length
l_{AW}	Axial length of the winding
l_{CL}	Winding conductor length lower bound
l_{CU}	Winding conductor length upper bound
l_{EL}	Lower bound for the core limb length
l_{EU}	Upper bound for the core limb length
l_{TL}	Transformer tank overall length
l_{TW}	Transformer tank overall width
l_{YL}	Lower bound for the core yoke length

- l_{YU} Upper bound for the core yoke length
- l_c Laminated core length

 l_w Winding length

- *n* Number of lumped parameter model sections per winding
- q(t) Net charge per capacitance tree branch cutset
- t_p Twice the paper insulation thickness
- v_{HR} Nameplate voltage for high voltage side
- v_{SC} Terminal voltage at rated current during a short circuit test
- w Angular frequency
- w Disc conductor width
- y Low voltage star connection

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Abstract

A power transformer will yield a frequency response which is unique to its mechanical geometry and electrical properties. Changes in the frequency response of a transformer can be potential indicators of winding deformation, as well as other structural and electrical problems. A diagnostic tool which can be used to detect such changes is Frequency Response Analysis (FRA). To date, FRA has provided only limited insight into the underlying physical cause of the change. There is now a growing research interest in identifying the physical change in a transformer directly from its FRA signature. The aim of the research in this thesis is to support the physical interpretation of FRA through the development of a wide-band three phase transformer model. The resulting model can be used for parameter sensitivity analysis, hence providing greater insight into the effects geometric change can have on transformer FRA. The research validates the proposed modelling approach by fitting the model to FRA data, without a priori knowledge of the internal dimensions, and then quantitatively assessing the accuracy of key geometric parameters. Finally, the ability of the model to support the interpretation of FRA is demonstrated. This is achieved by modifying a power transformer to emulate winding deformation and using the model to detect and quantify the degree of change.

Chapter 1

Introduction

1.1 History of Transformer Modelling

Modelling of transformers began at the start of the twentieth century. Abetti [9] suggests that the origins of transformer modelling can be traced back to a 1902 paper by Thomas [113]. In this paper Thomas observed that the "more abrupt the spark and the greater the capacity and inductance of the coils, the fewer the number of layers which will become charged before full terminal potential is reached, and the more severe will be the strain on insulation". Thomas' observation highlighted the need for modelling in order to determine the surge voltage distribution across a transformer's windings. In the discussion section of a paper by Jackson in 1906 [58], Steinmetz argued that at high frequencies a winding can be considered to respond more like a capacitor than an inductor. Blume and Boyajian in 1919 [28] considered the influence of a transformer's mutual and leakage inductance, the capacitance between adjacent coils, and the capacitance between the winding and ground. The Blume and Boyajian model is shown in Figure 1.1.

An alternative modelling approach proposed by Abetti in 1953 [8] developed a scale geometric model of a transformer core and windings. The scale model could be constructed such that it was representative of a large power transformer in terms of self and mutual inductances. However, the scale model could not emulate the capacitive elements of a power transformer and required the addition of external capacitors throughout the winding structure. Figure 1.2 shows a version of Abetti's scaled electromagnetic model for a single phase 667kVA transformer. This modelling approach had the limitation that with each variation in design, a new scale model would need to be constructed requiring non-trivial amounts of time and money [76].

The power of digital computers began to be harnessed in 1956 when McWhirter et al. determined the impulse voltage stress within a transformer winding using a lumped parameter model. McWhirter's group used the computational power provided by an IBM 650 digital card programmed calculator and an Electronic Associates 16-31R analog computer [77]. It was not until the late 1970's that transformer modelling reached a point considered adequate by industry. However these models were linear and lossless [52].



Figure 1.1: Early transformer model by Blume and Boyajian [28].



Figure 1.2: Abetti's 1953 electromagnetic transformer model [8].

In the late 1980's, early 1990's, articles by Wilcox et al. developed transformer models using modal analysis which, unlike previous work, took into account the frequency dependent effects and losses of a transformer core [122, 123, 124, 125]. However it was not until 1994 that de Leon and Semlyen published the first detailed three phase transformer model that incorporated non-linear and frequency dependent effects [35, 52].

Power transformer modelling still remains an active research area with over 200 journal articles published in IEEE on the topic within the last five years (2005 - 2010). With the continuous evolution of transformer models has also come additional areas of application, e.g. the interpretation of Frequency Response Analysis, which is discussed in the next section.

1.2 Applications of Power Transformer Models

An accurate transformer model is an important tool for use in the study of power transformers. This section discusses a number of practical applications where such models are utilised.

1.2.1 Winding Transient Analysis

For manufacturers to produce a robust yet cost effective transformer, it is important to accurately determine the transient voltage stress to which sections of the winding structure may be exposed. Transient simulation is typically based on the injection of standardised voltage impulses into a geometrically representative electromagnetic model of the transformer [76].

1.2.2 System Transient Analysis

Another area of application, and one that is employed by power utilities worldwide, is the use of transformer models in network transient and fault studies [50]. The utility will generally employ an electromagnetic transient program (EMTP) to study the complicated interaction between the various components in their electrical network [114]. In fact this is an area that is currently receiving much attention due to an increase in the number of transformer dielectric failures [2]. The Cigre working group A2/C4.39 has been formed to study the electrical transient interaction between transformers and the power system. One of this working group's key objectives is to investigate how high frequency transformer models used in EMTP can be improved [2].

1.2.3 Partial Discharge Localisation

A partial discharge (PD) is the breakdown of an insulation barrier within a system which results in an exchange of charge. The continual degradation of an area within the insulation system due to PDs could lead to a complete failure of the insulation integrity. This could have catastrophic consequences. As a result, there is clearly a need for regular monitoring of PD activity. The rate of degradation of a dielectric material can be directly correlated with the dissipated discharge energy. Therefore the use of PD pulse amplitude monitors has been common practice since the 1960's [22, 67]. However, a journal article by R.E. James in 1970 proposed that the terminal measurement of a PD has little value without knowledge of its originating location due to the possibility of high levels of attenuation within the windings. As a result, a variety of approaches to PD location have been tried over the years. Modern approaches include electrical modelling [118, 84], as well as acoustic and radio frequency triangulation [111]. The use of a lumped parameter electrical model for use in PD localisation is an area that has been investigated by the author [79, 80].

1.2.4 Frequency Response Analysis

Frequency response analysis (FRA) is a diagnostic tool which can be used to detect power transformer winding deformation [40]. It is also used to assess the mechanical condition after a transformer has been transported from one location to another. An FRA test involves the injection of a signal into one terminal whilst measuring the response at another. This procedure facilitates the determination of the frequency response "signature" between the various terminals on the power transformer. Ideally the comparison is between a recent FRA measurement and a previously recorded measurement for the same test configuration. However it is also useful to compare measurements of different terminal combinations on the same transformer, or of transformers of the same design.

There are a number of areas where transformer modelling can be used in conjunction with FRA. The first area is the need to quantify the degree of change in the frequency response relative to previously measured FRA data. Currently, this assessment is done by trained personnel and can be quite subjective due to the human element. The Cigre A2.26 working group suggests that the assessment process could be automated through comparison of mathematical models representative of the FRA measurements [6].

The second FRA modelling application is an extension of the first, and is directly applicable to the Cigre working group A2/C4.39 objectives discussed in Section 1.2.2. The objective is to use a mathematical model of a transformer which was derived from its FRA, as a component of the EMTP system model. This approach will improve the overall system model and assist in understanding the interactions between a transformer and the network [6].

The third application for FRA modelling, and the focus of this thesis, is the use of a transformer model as a tool to aid in the physical interpretation of changes in the FRA [6]. The rationale behind this approach is that a change in the geometry of a transformer will affect physical parameters represented in the transformer model. A sensitivity analysis of the model parameters could therefore be used to assist in determining the root cause behind any change in a transformer's frequency response.

1.3 Types of Transformer Models

Transformer models can generally be classified into one of two categories [116]. The first is the Black Box modelling approach. The goal of this model is to provide the time and/or frequency response characteristics relative to the transformer terminals. This type of model is generated by mathematical techniques in order to obtain an estimate of a transformer's transfer function [91]. The Black Box approach does not attempt to take into account the physical attributes of the transformer under test. The primary concern is the model accuracy as determined by a mathematical measure. A Black Box model can be used to accurately quantify the degree of frequency response variation between different FRA tests[120]. The second category of transformer model is the physical (or white box) model. A physical model is designed to emulate the complex electrical relationships that correspond to the structural and material characteristics of the transformer. This type of model facilitates a deeper understanding of an observed response, and can be useful in fault diagnosis [7]. Physical models can be classified as one or a combination of the following categories.

1.3.1 Transmission Line Model

The switching of gas-insulated switchgear can generate very fast transient over-voltages (VFTO). VFTOs can have a frequency content well into the megahertz range [72]. A lumped parameter modeling approach will not suffice at such high frequencies due to travelling wave effects where the windings behave more like transmission lines. It has been proposed that at these frequencies the winding conductors can be considered as waveguides to an electromagnetic wave [100]. As a result, for high frequency applications, an approach known as multi-conductor transmission line theory (MTL) can be used. MTL theory treats the entry of each winding junction as multiple parallel transmission lines where an incident transient is coupled into all paths [68].

1.3.2 Leakage Inductance Model

The windings of a transformer are tightly coupled. At low frequencies where the permeability of the core will still have a significant influence, the self and mutual inductance terms are nearly equal. As a result, an inductance matrix comprised of a winding's distributed self and mutual inductance terms will suffer from ill conditioning which makes inversion difficult [43]. Leakage inductance represents the difference between the self and mutual inductance of a winding section [92]. Since leakage inductance can be readily obtained from short circuit tests, the leakage inductance modelling approach uses the values determined through short circuit testing to generate the inverted inductance matrix directly [77].

1.3.3 Principle of Duality Model

The magnetic circuit of a transformer will have a significant influence on its low frequency response. As a result, for the study of the low frequency response it is important to have an accurate model of the transformer's magnetic circuit [128]. To achieve this, a common modelling approach is to apply the principle of duality as originally introduced by Cherry in 1949 [33]. The duality model derives an electric circuit based on a transformer's lumped parameter magnetic circuit and associated windings [102]. By constructing a model directly from its magnetic circuit, this approach facilitates the accurate modelling of the iron core [35].

1.3.4 Electromagnetic Field Model

The electromagnetic field modelling approach is based on the use of finite element analysis software to develop a comprehensive, three dimensional, electromagnetic model of a transformer. This approach is primarily adopted by manufacturers for use in the design stage of large transformers. An electromagnetic field model is computationally expensive and for most diagnostic applications it would not be practical [35].

1.3.5 Geometric Resistance Inductance Capacitance Model

This type of model is realised through the combination of self and mutual inductance, resistance, and capacitance, in order to emulate the electrical behaviour of a transformer's geometry. Non-linear and frequency dependent effects associated with a transformer core and windings can also be included [91]. Due to the physically representative nature of the model and wide-band accuracy (typically around 1MHz), it is the most widely adopted approach [93]. The model is useful for the calculation of branch currents and nodal voltages, and since it is geometrically representative, it is particularly useful for applications which require fault localisation such as FRA and PD [91]. It is for these reasons that the research in this thesis is based on this type of model.

1.4 Problem Statement, Motivation and Objectives

1.4.1 Problem Statement

Fault currents in a power transformer subject the windings and associated mechanical structure to high levels of mechanical stress. This stress can lead to winding deformation and hence, potentially, to transformer failure [40]. The winding deformation will result in subtle changes to the inductive and capacitive relationships of the winding. As a result, the transformer frequency response is altered and hence can be detected.

Transformer Frequency Response Analysis as proposed by Dick et al. in 1978 [40], is a commonly used tool for monitoring winding deformation in a power transformer. An FRA test injects a swept frequency sine wave between terminals of a transformer and calculates the resulting magnitude and phase response versus frequency. Generally, it is industry practice for trained personnel to visually compare the frequency response with historical records, or different phases in the same apparatus, or the same phase on sister units. Variation of the comparative responses may indicate a geometric change which can be indicative of structural damage. However, generally there is little understanding of the actual underlying cause or location of the change.

1.4.2 Motivation

To address the issue of FRA interpretation, the Cigre working group WG A2.26 for the *Mechanical Condition Assessment of Transformer Windings Using Frequency Response Analysis (FRA)*, has recommended further investigation [6] to improve FRA interpretation through,

"Transformer modelling based on geometrical parameters as a means to support the interpretation and derive a fundamental understanding of the FRA resonances..."

The research undertaken in this thesis is targeted at the Cigre recommendation by developing more comprehensive and flexible transformer models that will facilitate improved interpretation of FRA.

1.4.3 Current State of the Art

A number of researchers have developed detailed physical transformer models for application to FRA. Some of the key achievements in this area over the last decade are as follows:

• In 2000 Islam [56] proposed that an FRA spectrum can be partitioned into three distinct frequency ranges; low, medium and high. Using a ladder network model for the high voltage winding, the series capacitance was neglected for the low frequency region and the inductance was neglected for the high frequency region. An

investigation was then conducted into FRA sensitivity to transformer parameter change.

- In 2003 Rahimpour et al. [91] proposed a detailed single phase transformer model in order to diagnose axial displacement and radial deformation. The model included frequency dependent resistances for dielectric loss as well as winding conductor skin and proximity effects. Rahimpour et al. [91] assumed that the core losses and contribution to inductance would be negligible above 10kHz. The model parameters were estimated by both analytical and finite element calculations. They also investigated a range of FRA connection conditions in order to study the connection sensitivity.
- In a 2005 Bjerkan et al. [27] proposed a detailed three phase transformer model. This model included frequency dependent losses due to eddy current effects in the core and windings, as well as dielectric losses in the insulation. A two dimensional finite element analysis (FEA) model of the transformer winding was used to determine parameter values. One of the observations of Bjerkan's FEA simulation was that the inductance is still significant at 1MHz. A limitation of the two dimensional FEA model was the exclusion of three dimensional effects such as the coupling between phases. As a result the model was not applicable for low frequencies (<10kHz) [27].
- In 2006 Jayasinghe et al. [61] conducted research on the sensitivity of FRA measurement connections and their ability to detect different types of faults. The work demonstrated that no individual FRA test connection was best, it was fault dependent. Of all of the FRA tests conducted, the High Voltage End to End FRA test was the most sensitive to axial bending and the Capacitive Interwinding FRA test was the most sensitive to axial displacement and radial deformation. The research concluded by recommending that both End to End and Capacitive Interwinding FRA tests be conducted to ensure that all of the major fault types, for winding displacement, would be covered¹.
- Articles by Abeywickrama et al. between 2005 and 2008 [12, 13, 14, 16] provided a comprehensive three phase transformer model which used three dimensional FEA to derive the electrical model parameters. The model also included frequency dependent effects of the core, windings and insulation. Their model correlated quite well for frequencies between 100Hz and 1MHz for open circuit and primary to secondary impedance measurement tests. In [13], Abeywickrama et al. noted the diverging view amongst researchers regarding the frequency at which the contribution of the core to the winding inductance can be considered negligible. They proposed [13] that this frequency was above 1MHz.

¹For the sake of clarity, the FRA test connection names used throughout the thesis are as defined in [6]. The FRA test connections are discussed in detail in Chapter 7.

1.4.4 Research Objectives

During the course of the review in Section 1.4.3 it was apparent that a number of key areas are in need of further research. These areas are:

• The frequency at which the contribution of the core to the winding inductance can be considered negligible.

Recent articles have disagreed on this point with suggestions of 10kHz in [91] and 1MHz in [13, 5]. The researchers [13, 5] have used FEA simulation techniques to determine this frequency limit. A key objective in this thesis is to determine experimentally how the permeability of a power transformer's core varies with frequency. This will provide a practical confirmation of the frequency at which the contribution of the core to inductance can be considered negligible.

• The estimation of key model parameters without detailed knowledge of a power transformer's internal dimensions.

The latest research almost exclusively uses FEA to determine model parameters based on detailed knowledge of the internal dimensions of the transformer. These dimensions are rarely available to utilities or testing authorities due to intellectual property restrictions imposed by the manufacturer. An objective of this work is to develop a modelling approach that can estimate key physical parameters without detailed knowledge of a transformer's internal dimensions.

• Model fitting to multiple FRA configurations across all terminal permutations.

The research by Jayasinghe et al. [61] highlighted the importance of End to End and Capacitive Interwinding FRA in order to cover all major fault types. An objective of this thesis is to develop a flexible modelling platform that can be used for multiple FRA test configurations and include all the associated terminal permutations of each test.

• FRA interpretation of winding deformation through parameter change.

The research by Jayasinghe et al. [61] and Karimifard et al. [65] demonstrated how deformation in transformer windings could be simulated via parameter value changes in their model. An objective of this thesis is to take FRA interpretation a step further by replicating a winding deformation in a transformer and then verifying that particular model parameter estimates would correctly change to reflect the winding deformation. This objective will demonstrate the physically representative nature of the transformer model and its potential to support FRA interpretation.

It is the intention of this thesis to address each of the above objectives whilst developing a comprehensive three phase transformer model that can be used as a platform for the interpretation of FRA.

1.5 Thesis Overview

The objective of this thesis is to develop a three phase transformer model based on geometric parameters to facilitate the interpretation of FRA. Furthermore, since the internal dimensions of a transformer are generally not available due to intellectual property restrictions, techniques have been developed to estimate and constrain a number of key internal parameters to help ensure that the resulting transformer model is physically representative. To satisfy this objective, the thesis is structured in the following manner.

Chapter 2 presents a brief review of transformer design and modelling to support later chapters. It includes a discussion on windings, core topologies, transformer vector group connections, as well as basic transformer modelling theory for both single and three phase systems.

Chapter 3 experimentally determines the effective permeability bandwidth of a power transformer, and derives a permeability relationship for use within the transformer model. This chapter also investigates the effect that FRA test voltage levels will have on the core permeability. This investigation was necessary in order to confirm the compatibility of FRA derived from different FRA measurement instruments.

Chapter 4 derives the self and mutual inductance relationships for each winding on a three phase, double wound, core type transformer. The derivations are based on core permeability, geometry and dimensions, as well as the number of turns on each winding and an estimate of the leakage inductance.

Chapter 5 uses knowledge of typical transformer construction strategies in conjunction with transformer nameplate details, to derive an estimate of the leakage inductance, as required in the previous chapter.

Chapter 6 derives relationships for winding resistance including both skin and proximity effects. It also develops relationships for the capacitance between windings, windings and the core, windings and the tank walls, different phases, as well as across individual windings and discs. This chapter concludes by combining the resistance and capacitance, along with the self and mutual inductance developed in previous chapters, into a wideband generic phase model, i.e, a model that can be configured to represent the high and low voltage windings of phase A, B or C, for frequencies ranging from DC up to 1MHz.

FRA testing involves the injection and measurement of signals between various terminals on a transformer. This results in a large number of terminal permutations which need to be considered during modelling. The generic phase model developed in the previous chapter provides the flexibility necessary to construct an FRA model which is representative of an FRA test on any transformer vector group. On this basis, Chapter 7 develops models for three different types of FRA test conducted on transformers with a Dyn topology.

It is important that the estimation algorithm used with the models developed in the previous chapter can converge to a parameter solution set that is not just mathematically sound, but physically feasible. In order to facilitate this objective, Chapter 8 formulates

1.5. THESIS OVERVIEW

equations to provide initial parameter estimates and tight constraints on several key model parameters.

To confirm the veracity of the modelling approach developed in this thesis, Chapter 9 applies the models to test data which was obtained from a distribution transformer. The estimation algorithm determines the transformer parameters by simultaneously fitting each of the models to its corresponding FRA data set. Several key parameter estimates are then compared with their physically measured counterparts to demonstrate the modelling accuracy.

Chapter 9 also demonstrates how the models developed in this thesis could be used to support the interpretation of FRA. This is accomplished by modifying a distribution transformer in order to emulate a LV winding deformation, and then applying the parameter estimation algorithm in order to identify the key parameter changes specific to the induced fault.

The conclusion to this research is given in Chapter 10 including a summary of the results achieved and a discussion on future research.

The thesis also includes a number of appendices. Appendix A is a detailed discussion on inductive disparity. It examines how the difference in inductance between phases together with different vector group and measurement topologies, can produce a significantly different frequency response.

Appendix B justifies the assumption that the eddy current losses of the single layer winding used in Chapter 3 are small relative to core losses.

Appendix C gives a brief overview of Scattering (S) Parameters to expand on a discussion in Chapter 3.

Appendix D provides the two dimensional finite element analysis models which were used to benchmark the accuracy of the transformer model estimates for capacitance in Chapter 9.

1.6 Key Contributions

- Demonstrated experimentally that the effective complex relative permeability within a power transformer core will remain significant beyond 1MHz and is greater than unity at frequencies beyond 15MHz. When considered in the context of FRA, this implies that the core's complex relative permeability needs to be considered across the entire FRA test spectrum (<10MHz).
- Demonstrated experimentally that an FRA of a power transformer could be considered as a low field condition where the relative permeability of the transformer core approaches the initial permeability. This implies that, despite the highly non-linear nature of the core's hysteresis curve, there is a degree of independence with respect to the applied FRA test voltage.
- Developed estimates for the number of turns in each winding, transformer core yoke and limb dimensions, core cross sectional area, winding conductor cross sectional area and winding leakage inductance. These estimates are all based on the transformer nameplate, routine test data and external tank dimensions. They are used to place constraints on the parameters in the estimation algorithm such that the resulting parameter solution set is physically feasible.
- Developed a comprehensive wide-band frequency model of a three phase transformer for High Voltage Winding End to End Open Circuit, Low Voltage Winding End to End Open Circuit and Capacitive Interwinding FRA tests.
- Improved the accuracy of parameter estimates through the inclusion of multiple FRA data sets into the estimation algorithm. In the applied example, nine independent data sets produced as a result of End to End and Capacitive Interwinding FRA tests, were simultaneously used to estimate the model parameters.
- Demonstrated how a physically representative model of a power transformer could be used to identify winding deformation by looking at the relative change in key model parameters.
- Illustrated how the inherent inductive disparity between windings and the transformer's vector group will influence the resulting low frequency response.

1.7 Publications

The research embodied in this thesis has resulted in the following publications.

Journal Papers

- Steven D. Mitchell and James S. Welsh. "Modelling Power Transformers to Support the Interpretation of Frequency Response Analysis". *IEEE Transactions on Power Delivery*, (Approved for publication July 2011).
- Steven D. Mitchell and James S. Welsh. "The Influence of Complex Permeability on the Broadband Frequency Response of a Power Transformer". *IEEE Transactions* on Power Delivery, PP(99):1 –1, 2009.

Conference Papers

- Steven D. Mitchell, James S. Welsh. "Estimation of Physical Transformer Parameters from Frequency Response Analysis". *IEEE Power and Energy Society conference, PowerTech 2011*, Trondheim, Norway, June 2011.
- Steven D. Mitchell, James S. Welsh and Gustavo H.C. Oliveira. "Physical Modelling of Power Transformers Utilising End to End Frequency Response Analysis". *Cigre International Workshop on Power Transformers*, Foz do Iguacu, Brazil, April 2010.
- Gustavo H. C. Oliveira, Steven D. Mitchell, James S. Welsh, "Modeling Approaches Based on FRA Tests for Simulating and Analyzing a Power Transformer's Dynamic Behavior: A comparison", *Advanced Research Workshop on transformers*, *ARWtr* 2010, Santiago de Compostela, Spain, October 2010.
- Steven D. Mitchell and James S. Welsh. "Permeability and its Influence on the Broadband Frequency Response of a Power Transformer". Asia-Pacific Power and Energy Engineering Conference, APPEEC2009, Wuhan, China, March 2009.
- Steven D. Mitchell, J.S.Welsh, and B.T. Phung. "Relating a Distribution Transformer's Connection Topology and the Influence of Inductive Disparity to the Observed Frequency Response". Universities Power Engineering Conference, AU-PEC'08. Sydney, Australia, 2008.

1.8 Other Publications Related to the Thesis

 Gustavo H. C. Oliveira, Felipe F. Schauenburg, Rafael Maestrelli, Angélica C. O. Rocha, Steven D. Mitchell, James S. Welsh, "SimPowerSystems for the Study of Transients in the Irapé - Araçuaí System with the Aim of using Black Box Transformer Models". *Cigre International Workshop on Power Transformers*, Foz do Iguacu, Brazil, April 2010.

- Steven D. Mitchell, James S. Welsh, and Richard H. Middleton. "A Narrowband High Frequency Distributed Power Transformer Model for Partial Discharge Location". Universities Power Engineering Conference, AUPEC 2007. Perth, Australia, Dec. 2007.
- James S. Welsh, Cristian R. Rojas, and Steven D. Mitchell. "Wideband Parametric Identification of a Power Transformer". Universities Power Engineering Conference, AUPEC 2007. Perth, Australia, Dec. 2007.
- 4. Steven D. Mitchell, James S. Welsh, Richard H. Middleton, and B.T. Phung. "Practical Implementation of a Narrowband High Frequency Distributed Model for Locating Partial Discharge in a Power Transformer". *Electrical Insulation Conference* and Electrical Manufacturing Expo, EIC2007, Nashville, U.S.A., Oct. 2007.
- Shuzhen Xu, Steven D. Mitchell and Richard H. Middleton. "Partial Discharge Localization for a Transformer Based on Frequency Spectrum Analysis". Universities Power Engineering Conference, AUPEC2003, Christchurch New Zealand, September 2003.

Chapter 2

Power Transformers: A Review

2.1 Introduction

This chapter provides a brief review of power transformers. The review begins with basic nomenclature, standards and definitions that are used throughout the thesis. It then proceeds to discuss the basics of transformer core construction and the more common core types found in power transformers. Transformer windings are then described, including their relative location on the core, common winding types, and typical conductor materials. An overview on insulation and cooling, highlighting the dual role played by mineral oil, is also provided. Basic transformer modelling is then discussed by introducing an ideal single phase transformer model. We then consider the effects of flux leakage and finite permeability. Finally, a single phase model is used to develop per phase equivalent circuits for the most common three phase transformer vector groups.

This chapter is structured in the following manner. Definitions and referencing used in the thesis are given in Sections 2.2 and 2.3. Transformer construction is described in Sections 2.4 through to 2.6. Finally, an overview of single and three phase transformer modelling is presented in Sections 2.8 and 2.9 with concluding remarks in Section 2.10.

2.2 Power Transformers

A transformer is a static electrical device that uses electromagnetic induction to transfer power from one circuit to another [52]. Power transformers are an essential component of a power system which are typically designed to have a 30-40 year operating life [71]. Their function is to transform voltages to suitable levels between the generation, transmission and distribution stages of a power system. Power transformers can be classified into three categories based on their power ratings; small (500 to 7500kVA), medium (7500kVA to 100MVA) and large (100MVA+) [52].

2.3 Referencing

As there are several international standards for referencing transformer terminal and vector groups, it is important to establish consistent nomenclature prior to any detailed discussion. For geographical reasons, we adopt the IEC60076-1 standard.

Throughout the thesis, short hand notation for the high and low voltage windings are HV and LV respectively.

2.3.1 International System of Units

Unless otherwise stated, the International System of Units (SI) are used for all units of measure in this thesis.

2.3.2 Phase Referencing

We use A-B-C as the phase reference characters. As is common in all standards, the high voltage winding is represented by an upper case character and the low voltage winding by a lower case character. For example, the HV terminals are designated ABC and the LV terminals, *abc*.

2.3.3 Generic Referencing

This thesis focuses on transformer modelling for FRA. Taking a High Voltage End to End FRA as an example¹ [6], one test connection permutation is to inject into the HV phase B terminal and record the response on the HV phase C terminal leaving the HV phase A and LV phase terminals open circuited. This test would be repeated for the other two phase permutations. To represent this example in a model, it is convenient to have generic phase references for the injection, measurement and open circuited terminals. This work accomplishes this through the use of generic phase referencing. Throughout the thesis the generic high voltage terminals are designated X-Y-Z, and the corresponding low voltage terminals are x-y-z.

2.3.4 Vector Group

Table 2.1 lists the transformer winding connections discussed in this thesis and their corresponding designator codes. The transformer vector group is the term given to combinations of the designator codes which specify the complete transformer connection topology. The vector group code also includes one or two digits. These digits specify the phase displacement between the windings and is discussed in the next section.

 $^{^{1}}$ FRA test connections are discussed in detail in Chapter 7
Winding	Winding Connection	Designator
HV	Delta	D
LV	Delta	d
HV	Star	Y
LV	Star	у

Table 2.1: IEC60076-1 winding connection designators

Reference Hour	Phase Displacement
0	0°
1	-30°
6	180°
11	$+30^{\circ}$

Table 2.2: Clock reference to phase displacement (IEC60076-1)

2.3.5 Phase Displacement

The vector group of a transformer has an effect on the phase displacement between the high and low voltage windings. The vector group details of a transformer will provide the necessary information to determine the phase displacement. This information is critical from a modelling perspective due to inductive disparity which is discussed in Appendix A.

To indicate the phase displacement between the high and low voltage windings, a parallel is drawn to an analog clock [53]. The minute hand represents the phase voltage of the HV winding and is positioned at 12 o'clock. The hour hand represents the phase voltage for the LV winding. Each "hour" represents 30 degrees phase displacement. As an example, transformers with a Dyn1 or Dyn11 vector group have a high voltage delta connection and a low voltage star connection. However, the Dyn1 vector group has a phase displacement of -30° and the Dyn11 vector group has a phase displacement of $+30^{\circ}$.

Table 2.2 relates the reference hour used in the transformer vector group number to its associated phase displacement for common power transformer configurations. The polarity of the phase displacement is dependent upon the terminal connection order.

Note that the vector group number (reference hour) will only be added if it is pertinent to the discussion.

2.4 Transformer Core

In this section we discuss the materials used in the construction of a transformer core and also the different core topologies which are in common use today.

2.4.1 The Core

A transformer utilises the low reluctance path provided by a magnetic core to transfer energy from one winding to another. To minimise losses within the core, it is typically constructed from laminated sheets of high grade silicon steel² [101]. The silicon in the steel increases resistivity which reduces eddy currents and hence reduces the associated losses. Silicon also increases the material permeability and reduces hysteresis losses. A disadvantage of the use of silicon is that it makes the steel harder and more brittle. Due to this disadvantage the silicon content is typically limited to a maximum of 4.5% to ensure material workability [53].

The silicon steel laminations are typically cold rolled in order to orientate the steel grain. This product is referred to as cold rolled grain oriented (CRGO) silicon steel. This has the effect of making the permeability in the rolled direction significantly greater than the permeability in the transverse direction [15]. The laminations can be further treated through laser or mechanical scribing. The scribing of the steel reduces power loss by introducing surface defects which refine the magnetic domains [119]. Lamination thickness ranges from 0.23mm to 0.35mm [71]. The choice of lamination thickness is generally a tradeoff between eddy current losses and transformer manufacturing costs. Smaller lamination thicknesses result in more laminations to be laid for a given core cross section which results in increased labour costs.

The saturation level for flux density in the core of a power transformer is approximately 2.0 Tesla [52]. For the transformer to operate efficiently, it is important that the flux density remain below the B-H curve "knee" (See Figure 2.1). For flux densities above the "knee", significantly greater magnetising currents are required for small increases in flux density. However, operating too far below the "knee" will not utilise the core to its full potential, and is therefore not cost effective. An example B-H curve is shown in Figure 2.1. Typical peak operating flux densities range between 1.6 and 1.8 Tesla for CRGO silicon steel [20, 53].

The core is electrically insulated from the transformer's mechanical structure except for a single ground point. The ground point provides a path to dissipate accumulated electrostatic charge. Additional ground points would facilitate circulating currents which would result in additional power loss and localised heating [53].

 $^{^{2}}$ The use of amorphous steel as a core material results in a significant reduction in core losses. However, due to the brittle nature of the material, manufacture is difficult and as a result its use has been somewhat limited.



Figure 2.1: B-H Curve for 3.5% silicon electrical steel at 50Hz (Data from [29]).

2.4.2 Types of Core

There are two main types of core construction used in power transformer manufacture, the shell and the core form. The shell form of construction provides a magnetic circuit that encloses the windings, forming a "shell". A three phase example of the shell form is shown in Figure 2.2(a). It has the advantage of a better short circuit and transient voltage response relative to the core form [52]. However, construction is more complex leading to greater manufacturing costs. As a result, the shell form tends to find application in large power transformers where severe operating conditions can exist. In the core form construction, the windings are wound concentrically around the core limbs. This construction approach is simpler, easier to repair, and has thermal property advantages over the shell form. As a result, it is the most common construction approach used in power transformer manufacture [66].

The three phase core form construction can be further divided into three limb and five limb designs as shown in Figures 2.2(b) and (c) respectively. For the three limb core design, each limb corresponds to one phase. The magnetic circuit is completed by joining each limb with a top and bottom yoke. For a balanced three phase system, the sum of fluxes within the three limb design will always be zero [53].

For very large transformers, transport height restrictions are an important design consideration. It is in such cases that the five limb core construction finds application. The provision of the two additional outside limbs provide an alternative flux path that enables a reduction in the top and bottom yoke cross sectional area. This reduction can be as much as 50% relative to the limb cross sectional area [53].





Figure 2.2: Three phase transformer core configurations. (a) Shell form, (b) three limb core form, (c) five limb core form.

2.5. TRANSFORMER WINDINGS



Figure 2.3: Continuously transposed conductor (CTC)

2.5 Transformer Windings

This section reviews the different materials used in the construction of a transformer winding. It then proceeds to discuss the different winding topologies and their respective orientation relative to the transformer core.

2.5.1 Conductor Material

A transformer winding consists of an insulated conductor wound around the transformer core. The conductor material is generally either copper or aluminium. Whilst aluminium is less expensive and lighter, it requires a larger cross sectional area to carry the same current as its copper equivalent. As a result, aluminium conductors are not as common [52].

Copper conductors are typically paper insulated with a rectangular section, although smaller power transformers may use foil or sheet conductors. The conductors can be arranged individually or as multiple strands in parallel. In the case of multiple stranding, voltage differences may induce circulating currents. To minimise this effect it is necessary to continuously transpose the conductor strands. This conductor topology is referred to as a continuously transposed conductor (CTC) and is shown in Figure 2.3.

2.5.2 Types of Windings

There are a number of different types of windings used in transformer construction. Their application is dependent upon the specific current and voltage levels, as well as the type of core construction. In this section the focus is on disc windings for the high voltage, and helix and layer based windings for the low voltage.

Disc Winding

An individual disc consists of single or multiple insulated conductors wound concentrically around one another. The discs are physically adjacent and are electrically connected in series (see Figure 2.4). Disc windings are commonly used for the high voltage windings of core form transformers (refer Figure 2.2(b) and (c)).



Figure 2.4: Common disc winding strategies

In the event of a large transient voltage, high levels of electrical stress can be experienced in the discs that are closest to the input. By increasing the overall series capacitance of the winding, the transient voltage distribution is more uniform and therefore places less stress upon individual discs. An increase in the series capacitance of the winding can be achieved in a number of ways. One approach is to interleave the turns within the disc. An alternative approach is to introduce an electrostatic shield between the discs. Both of these approaches, along with the standard continuous disc topology, are shown in Figure 2.4. In this figure the sequence of turns is indicated numerically.

Helix Winding

The helix winding is constructed by winding insulated conductors in parallel, in a corkscrew fashion. The number of conductors in parallel can range from a few to over 100 [52]. To minimise the effects of circulating currents in the parallel conductors, it is necessary to introduce winding transpositions. This type of winding is typically used for high current, low voltage applications.

Layer Winding

The layer winding is one of the simplest forms of construction. It involves the continuous winding of an insulated conductor along the length of the winding cylinder and can include multiple conductors wound in parallel. This process is repeated to obtain several separate winding layers.

2.5.3 Winding Location

Double wound power transformers have found widespread application [53]. A double wound power transformer consists of a high and a low voltage winding on each phase. The LV winding is typically located closest to the core. The HV winding is then placed concentrically around the LV winding. These positions are based on two key factors. The first is the lower level of insulation required to insulate the LV winding from the earthed core. The second factor is that the transformer taps are typically placed on the HV winding, and will therefore be more accessible. A cross section of a core limb for a double wound transformer construction is shown in Figure 2.5.

2.6 Transformer Insulation and Cooling

The insulation system of a power transformer is typically constructed from a combination of paper and pressboard cellulose material which is immersed in mineral oil. The oil impregnated cellulose material is low cost and has excellent insulation properties. It is used to insulate winding turns and is also formed into insulating cylinders and barriers to separate winding sections from each other, and the earthed core.



Figure 2.5: Core limb cross section of a double wound transformer.

The mineral oil has a dual purpose. It not only provides a highly effective insulation barrier, but it also facilitates the removal of thermal energy from the transformer. Convection effects within the transformer tank will generate natural oil circulation which results in cooling due to the thermal energy in the oil being transferred to the tank walls. In large transformers, this effect can be supplemented through the addition of oil pumps. The heat exchange can be further improved by adding external fans and also by increasing the heat transfer area through the addition of external radiators.

2.7 Transformer Modelling Assumptions

For the purpose of the research in this thesis we focus on small to medium sized power transformers due to the availability of data and testing facilities for experimental verification (Section 2.2). As a consequence, the three limb core form design is considered since it is the most common construction approach for small and medium sized power transformers [71] (Section 2.4). It is also assumed that the transformer is double wound with a disc winding on the high voltage side and either a helix or layer based winding on the low voltage side (Section 2.5).



Figure 2.6: Basic electromagnetic principles of a single phase transformer

2.8 Basic Single Phase Transformer Model

2.8.1 Ideal Transformer

In order to construct a wide-band frequency model of a power transformer, it is important to understand the fundamental relationships. This section develops a model of a single phase transformer operating at its rated frequency. This background material is presented here for completeness and also to develop some of the nomenclature that is used throughout the remainder of the thesis.

With reference to Figure 2.6, the application of a voltage v_1 to the primary winding causes a current, i_1 , to flow into the primary winding. This current produces a magnetomotive force \mathcal{F}_1 that is proportional to the number of turns of the winding, N_1 [89], i.e.

$$\mathcal{F}_1 = N_1 i_1 \quad . \tag{2.1}$$

Neglecting leakage flux, for the moment, the resulting magnetomotive force produces a flux Φ_m in the magnetic core of a transformer. Since Φ_m links both the primary and secondary windings, it is referred to as the mutual flux. If v_1 is time varying, Φ_m will also vary with time and an electromotive force, e_1 , is induced into the primary winding such that,

$$e_1 = N_1 \frac{d\Phi_m}{dt} \quad . \tag{2.2}$$

According to Lenz's law, the polarity of the induced electromotive force must be in a

direction such as to oppose the change in flux causing it. In this case, assuming that i_1 is increasing with respect to time, e_1 has the reference polarity as shown in Figure 2.6. Since Φ_m links the secondary winding, an electromotive force e_2 is induced. Maintaining that i_1 is increasing with respect to time, the induced electromotive force e_2 has a polarity opposed to the change in flux and produces a current i_2 through the load Z [24]. The electromotive force e_2 can now be defined as,

$$e_2 = N_2 \frac{d\Phi_m}{dt} \quad . \tag{2.3}$$

Combining equations (2.2) and (2.3),

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} \ . \tag{2.4}$$

Neglecting copper losses and flux leakage, the transformer voltage to turns ratio relationship is,

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = \bar{a} , \qquad (2.5)$$

where \bar{a} is referred to as the transformer turns ratio or ratio of transformation³. With a current i_2 flowing through a winding of N_2 turns, an opposing magnetomotive force of \mathcal{F}_2 is developed such that,

$$\mathcal{F}_2 = N_2 i_2 \quad . \tag{2.6}$$

The resultant magnetomotive force in the transformer's magnetic core is the difference between the opposing magnetomotive forces,

$$\mathcal{F}_{\mathrm{m}} = \mathcal{F}_1 - \mathcal{F}_2 \quad . \tag{2.7}$$

In an ideal transformer, the permeability of the core is assumed to be infinite and the core assumed to have no losses. Therefore the magnetomotive force required to produce flux in the core will tend to zero and hence,

$$\mathcal{F}_1 = \mathcal{F}_2 \quad . \tag{2.8}$$

From (2.1), (2.6) and (2.8), the transformer current to turns ratio relationship is,

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{\bar{a}} \ . \tag{2.9}$$

2.8.2 Flux Leakage

In a non-ideal transformer, not all of the magnetic flux generated by a magnetomotive force \mathcal{F} is confined to the core. There is flux leakage around each winding such that some

³Throughout this thesis the transformer turns ratio is designated \bar{a} rather than a so as to provide a distinction between the turns ratio and the low voltage phase A designator.



Figure 2.7: Single phase transformer with flux leakage

flux bypasses the core and completes its path through the air [101]. The flux leakage of the primary and secondary windings, Φ_{L1} and Φ_{L2} respectively, are shown in Figure 2.7. Maintaining the reference direction of Figure 2.7, the respective flux linking the primary and secondary windings is,

$$\Phi_1 = \Phi_m + \Phi_{L1} , \qquad (2.10)$$

$$\Phi_2 = \Phi_m - \Phi_{L2} \ . \tag{2.11}$$

Due to flux leakage, the electomotive force relationships in (2.2) and (2.3) become

$$e_1 = N_1 \frac{d\Phi_m}{dt} + N_1 \frac{d\Phi_{L1}}{dt} \tag{2.12}$$

and

$$e_2 = N_2 \frac{d\Phi_m}{dt} - N_2 \frac{d\Phi_{L2}}{dt} \quad . \tag{2.13}$$

If the current in winding X is denoted i_X , then the self inductance L is given by [89],

$$L = \frac{\text{Flux linking circuit X due to } i_X}{i_X} . \tag{2.14}$$

The leakage inductance for the primary and secondary windings can therefore be defined as,

$$L_{L1} = \frac{N_1 \Phi_{L1}}{i_1} \tag{2.15}$$

and

$$L_{L2} = \frac{N_2 \Phi_{L2}}{i_2} , \qquad (2.16)$$

where L_{L1} is the primary winding leakage inductance and L_{L2} is the secondary winding leakage inductance.

2.8.3 Finite Permeability

A non-ideal transformer core also has finite permeability [101]. With finite permeability, the core exhibits reluctance \mathcal{R}_m such that the mutual flux flowing within the transformer core is given by,

$$\mathcal{F}_m = \Phi_m \mathcal{R}_m \quad . \tag{2.17}$$

With reference to Figure 2.7, assuming that the impedance Z is removed such that the secondary is open circuited, then $i_2 = 0$. In this case the magnetisation current is provided by the primary alone and is referred to as the primary magnetisation current, i_m . From (2.17),

$$\mathcal{F}_{\mathrm{m}} = N_1 i_m = \Phi_m \mathcal{R}_m \quad . \tag{2.18}$$

From (2.1), (2.6), (2.7) and (2.18),

$$N_1 i_m = N_1 i_1 - N_2 i_2 \quad . \tag{2.19}$$

Rearranging (2.19) and incorporating the transformer turns ratio (2.9),

$$i_1 = i_m + \frac{N_2}{N_1} i_2 = i_m + \frac{i_2}{\bar{a}}$$
 (2.20)

Using (2.14), the primary referred magnetising inductance L_m can be defined as,

$$L_m = \frac{N_1 \Phi_m}{i_m} \ . \tag{2.21}$$

In a similar fashion, the magnetising inductance could also be referred to the secondary winding.

2.8.4 Transformer Equivalent Circuit

The primary winding can be considered to have both a leakage inductance (2.15) and a primary referred magnetising inductance (2.21). In addition, a practical transformer also



Figure 2.8: Transformer equivalent circuit

has copper, or $i^2 R$, losses in each winding. The primary voltage v_1 can be defined in terms of the voltage drops across each of these elements,

$$v_1 = R_1 i_1 + L_{L1} \frac{di_1}{dt} + e_{m1} \quad , \tag{2.22}$$

where R_1 is the primary winding resistance and e_{m1} represents the magnetising electromotive force,

$$e_{m1} = N_1 \frac{d(\Phi_m)}{dt} = L_m \frac{di_m}{dt}$$
 (2.23)

In a similar fashion, taking into account the reference directions, the secondary winding equivalent circuit voltage is given by,

$$v_2 = -R_2 i_2 - L_{L2} \frac{di_2}{dt} + e_{m2} \quad . \tag{2.24}$$

With reference to (2.22) and (2.24) it is noted that to model the non-ideal properties of copper loss and leakage inductance, the ideal transformer model requires series resistance and leakage inductance. In addition, from equation (2.20), the primary winding current can be split into two different current branches, a magnetising inductance branch and an ideal current ratio branch. An equivalent circuit for these effects is given in Figure 2.8.

2.8.5 Simplified Equivalent Circuit

A power transformer core is typically constructed using grain oriented silicon steel laminations. As a result, the core at mains frequency has a high permeability. Due to the high permeability, the magnetising inductance can be viewed as a large shunt reactance. In contrast, the series copper loss and leakage reactance parameters are comparatively very small. Therefore a reasonable approximation can be made to refer the secondary series parameters to the primary side (or alternatively, the primary series parameters to the secondary side). This is accomplished by using the voltage and current gain relationships



Figure 2.9: Simplified transformer equivalent circuit

of an ideal transformer. From Figure 2.8,

$$\frac{e_{m1}}{e_{m2}} = \frac{N_1}{N_2} = \bar{a}$$

$$\therefore e_{m1} = \bar{a}e_{m2} . \qquad (2.25)$$

Since the voltage drop across the leakage and loss terms is small relative to the voltage drop across the magnetising inductance, an approximation to (2.25) is,

$$v_1 = \bar{a}v_2 \quad . \tag{2.26}$$

As the magnetising current is small relative to the load current, it can be assumed that $i_m \rightarrow 0$ and from (2.9),

$$i_1 = \frac{i_2}{\bar{a}}$$
 . (2.27)

Dividing (2.26) by (2.27),

$$\frac{v_1}{i_1} = \bar{a}^2 \frac{v_2}{i_2}$$

$$\therefore Z_1 = \bar{a}^2 Z_2 . \qquad (2.28)$$

Hence an impedance on the secondary side can be referred to the primary side by scaling with the turns ratio squared. This simplifies the transformer equivalent circuit of Figure 2.8 to that of Figure 2.9, where

$$R = R_1 + \bar{a}^2 R_2 , \qquad (2.29)$$

$$L_L = L_{L1} + \bar{a}^2 L_{L2} . (2.30)$$

This equivalent circuit proves useful in the following discussion on per phase equivalent circuits for three phase transformers.

2.9 Basic Three Phase Transformer Model

This section applies the relationships derived for the single phase transformer model to that of a three phase transformer operating at its nameplate frequency. The primary winding is assumed to be the high voltage winding. A positive phase sequence is assumed throughout these derivations.

2.9.1 Vector Group Gain

Under balanced conditions, a three phase transformer can be considered to be the interconnection of three single phase transformers. However, the vector group of a three phase transformer can influence its voltage and current gain [24]. This section analyses the four most common vector groups used in double wound three phase power transformers and discusses the vector group versus gain relationship for each.

Delta-delta (Dd) vector group

For the Dd vector group in Figure 2.10(a), there is a direct inductive coupling between the primary line to line voltage, v_{AB} , and the secondary line to line voltage, v_{ab} . With reference to (2.5),

$$\frac{v_{ab}}{v_{AB}} = \frac{N_x}{N_X} = \frac{1}{\bar{a}} \quad , \tag{2.31}$$

where N_X and N_x are a generic representation for the number of turns of the primary and secondary windings respectively. Hence there is a transformer voltage gain from the primary to the secondary of,

$$K_V = \frac{1}{\bar{a}} \quad . \tag{2.32}$$

Similarly, with reference to (2.9), the current gain is,

$$\frac{i_{ab}}{i_{AB}} = \frac{N_X}{N_x} = \bar{a} \quad , \tag{2.33}$$

where i_{AB} is the current in the primary winding between terminals A and B and i_{ab} is the current in the secondary winding between terminals a and b. Therefore, the current gain from the primary to the secondary is,

$$K_i = \bar{a} \quad . \tag{2.34}$$



Figure 2.10: Common vector groups used in double wound three phase power transformers

Star-star (Yy) vector group

The Yy vector group of Figure 2.10(d) also has a direct inductive coupling between the primary line to neutral voltage, v_{AN} , and the secondary line to neutral voltage, v_{an} .

$$\frac{v_{an}}{v_{AN}} = \frac{N_x}{N_X} = \frac{1}{\bar{a}}$$
 (2.35)

As with a Dd connection, the primary to secondary voltage gain of Yy is,

$$K_V = \frac{1}{\bar{a}} \quad . \tag{2.36}$$

Similarly, the current gain is,

$$\frac{i_{an}}{i_{AN}} = \frac{N_X}{N_x} = \bar{a} \quad , \tag{2.37}$$

where i_{AN} is the primary winding current and i_{an} the secondary winding current. Therefore the current gain between the primary and secondary is,

$$K_i = \bar{a} \quad . \tag{2.38}$$

Delta-star (Dy) vector group

For the Dy vector group, Figure 2.10(b), the primary line to line voltage v_{AB} inductively couples to the secondary line to neutral voltage v_{an} ,

$$\frac{v_{an}}{v_{AB}} = \frac{1}{\bar{a}} . \tag{2.39}$$

To find the voltage gain, it is necessary to have the voltages in the same form. Since,

$$v_{AB} = v_{AN} - v_{BN} = \sqrt{3} v_{AN} e^{j\pi/6} , \qquad (2.40)$$

then by substituting (2.40) into (2.39),

$$\frac{v_{an}}{\sqrt{3}v_{AN}\,e^{j\pi/6}} = \frac{1}{\bar{a}} \tag{2.41}$$

$$\therefore \frac{v_{an}}{v_{AN}} = \frac{\sqrt{3}}{\bar{a}} e^{j\pi/6} . \qquad (2.42)$$

Hence there is a connection based voltage change and a $+30^{\circ}$ phase shift resulting in a total voltage gain of,

$$K_V = \frac{\sqrt{3}}{\bar{a}} e^{j\pi/6} \quad . \tag{2.43}$$

For the current gain of the Dy vector group,

$$N_X i_{AB} = N_x i_{an} av{2.44}$$

The primary delta winding current in terms of the line current is,

$$i_A = i_{AB} - i_{CA} . (2.45)$$

Taking into account the turns ratio and the coupling between the primary and secondary windings, (2.45) is equivalent to,

$$i_{A} = \frac{1}{\bar{a}} (i_{an} - i_{cn}) = \frac{\sqrt{3}}{\bar{a}} i_{a} e^{-j\pi/6} .$$
 (2.46)

Therefore the current gain of a Dy connection is given by,

$$K_i = \frac{i_a}{i_A} = \frac{\bar{a}}{\sqrt{3}} e^{j\pi/6} \quad . \tag{2.47}$$

Star-delta (Yd) vector group

In the Yd vector group, Figure 2.10(c), the primary line to neutral voltage of the Star connection, v_{AN} , inductively couples to the secondary line to line voltage of the delta connection, v_{ab} ,

$$\frac{v_{ab}}{v_{AN}} = \frac{1}{\bar{a}} \quad . \tag{2.48}$$

Since,

$$v_{ab} = v_{an} - v_{bn} = \sqrt{3}v_{an} e^{j\pi/6}$$
 (2.49)

Substituting (2.49) into (2.48),

$$\frac{\sqrt{3}v_{an}\,e^{j\pi/6}}{v_{AN}} = \frac{1}{\bar{a}} \tag{2.50}$$

$$\therefore \frac{v_{an}}{v_{AN}} = \frac{1}{\bar{a}\sqrt{3}}e^{-j\pi/6} . \qquad (2.51)$$

Hence, there is a connection based voltage change and a -30° phase shift for the voltage gain,

$$K_v = \frac{1}{\bar{a}\sqrt{3}}e^{-j\pi/6} \quad . \tag{2.52}$$

2.9.	BASIC	THREE	PHASE	TRANSF	ORMER	MODEL

Vector Group	Voltage Gain (K_V)	Current Gain (K_i)
Dd	$\frac{1}{\bar{a}}$	\bar{a}
Dy	$rac{\sqrt{3}}{ar{a}} e^{j\pi/6}$	$rac{ar{a}}{\sqrt{3}} e^{j\pi/6}$
Yd	$\frac{1}{\bar{a}\sqrt{3}}e^{-j\pi/6}$	$\bar{a}\sqrt{3}e^{-j\pi/6}$
Yy	$\frac{1}{\overline{a}}$	\bar{a}

Table 2.3: Vector group based voltage and current gain for three phase transformers

For the current gain of the Yd vector group,

$$N_X i_{AN} = N_x i_{ab} \quad . \tag{2.53}$$

The delta winding current in terms of the line current is,

$$i_a = i_{ab} - i_{ca}$$
 . (2.54)

Taking into account the turns ratio and the coupling between the primary and secondary windings, (2.54) is equivalent to,

$$i_{a} = \bar{a} (i_{AN} - i_{CN}) = \bar{a} \sqrt{3} i_{A} e^{-j\pi/6} .$$
(2.55)

Therefore the current gain for a Yd vector group is given by,

$$K_i = \frac{i_a}{i_A} = \bar{a}\sqrt{3} \, e^{-j\pi/6} \quad . \tag{2.56}$$

A summary of the three phase transformer gains is presented in Table 2.3. An important point to note is that the current gain is equal to the inverse of the complex conjugate of the voltage gain [24],

$$K_i = \frac{1}{K_V^*}$$
 (2.57)

2.9.2 Delta to Star Transformation

It is often necessary, for the purpose of model simplification, to convert between a delta and a star connection. With reference to the delta connection of Figure 2.11,

$$i_a = i_{ab} - i_{ca} = \frac{v_{ab}}{Z_D} - \frac{v_{ca}}{Z_D}$$
 (2.58)

This can be rearranged such that,

$$Z_D = \frac{v_{ab} - v_{ca}}{i_a} \ . \tag{2.59}$$

Similarly, with reference to the star connection,

$$v_{ab} = i_a Z_Y - i_b Z_Y = Z_Y (i_a - i_b)$$
(2.60)

and

$$v_{ca} = Z_Y(i_c - i_a) .$$
 (2.61)

Combining (2.60) and (2.61),

$$v_{ab} - v_{ca} = Z_Y(2i_a - i_b - i_c) . (2.62)$$

From Kirchhoff's Current Law, the sum of all currents must be equal to zero, i.e.

$$i_a + i_b + i_c = 0 . (2.63)$$

Rearranging (2.63) in terms of i_a and substituting into (2.62),

$$v_{ab} - v_{ca} = 3i_a Z_Y \tag{2.64}$$

$$3Z_Y = \frac{v_{ab} - v_{ca}}{i_a} \ . \tag{2.65}$$

Equating (2.59) with (2.65),

$$Z_D = 3Z_Y \quad . \tag{2.66}$$

From (2.66), it is observed that under balanced conditions a transformation can be made between delta and star connections. This relationship is an integral part of the per phase analysis of section 2.9.3.

2.9.3 Per Phase Equivalent Circuit Analysis

Under balanced conditions, it is possible to reduce the complexity of the system model for various three phase power transformer connections by considering them on a per phase basis [24]. In this section, a per phase equivalent circuit is developed for each of the four vector groups considered thus far. For the sake of simplicity, winding resistance losses are neglected.

Yy vector group

With reference to Figure 2.12, it is apparent that under balanced conditions, each phase of the Yy vector group can be modeled by a single phase equivalent circuit. By considering the voltage and current gains of (2.36) and (2.38) and with reference to Figure 2.8, a

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Figure 2.11: Delta Star transformation

per phase equivalent circuit can be produced. To be phase non-specific, generic phase X is utilised. The per phase equivalent circuit for the Yy vector group is presented in Figure 2.14(d).

Dd vector group

By utilising the Delta to Star impedance transformation from (2.66), under balanced conditions the delta winding can be converted to a star connection by dividing each impedance by a factor of 3. The Dd vector group can then be converted into Yy form as shown in Figure 2.13. Once in this form and taking into account the respective voltage and current gains from (2.32) and (2.34), each phase can be represented by the equivalent circuit of a single phase transformer. The Dd per phase equivalent circuit using generic phase X is shown in Figure 2.14(a).

Dy vector group

The Dy vector group is not as straight forward as Dd and Yy connections. In addition to the delta to star impedance conversion required for the primary delta winding, there is also the connection based voltage and current gains of (2.43) and (2.47). The Dy per phase equivalent circuit is shown in Figure 2.14(b).

Yd vector group

For the Yd vector group, the secondary delta connection is converted to star and the connection based voltage and current gains are given by (2.52) and (2.56). The Yd vector group per phase equivalent circuit is shown in Figure 2.14(c).



Figure 2.12: Yy equivalent circuit



Figure 2.13: Dd vector group converted to a Yy equivalent circuit



Figure 2.14: Per phase equivalent circuit topologies

2.9.4 Simplified per Phase Equivalent Circuit

A simplified per phase equivalent circuit can be developed by referring the secondary leakage inductance and resistance to the primary side. In the same manner as the single phase derivation in Section 2.8.5, the voltage and current gains need to be considered. For the (generic phase) voltage gain,

$$\frac{v_{xn}}{v_{XN}} = K_V$$

$$\therefore v_{XN} = \frac{v_{xn}}{K_V} . \qquad (2.67)$$

For the (generic phase) current gain,

$$\frac{i_{XN}}{i_{XN}} = K_i$$

$$\therefore i_{XN} = \frac{i_{Xn}}{K_i} . \qquad (2.68)$$

Combining (2.67) and (2.68),

$$\frac{v_{XN}}{i_{XN}} = \frac{K_i v_{xn}}{K_V i_{xn}}$$

$$Z_X = Z_x \left[\frac{K_i}{K_V} \right] .$$
(2.69)

Noting the current to voltage gain from (2.57),

$$Z_X = Z_x \frac{1}{|K_V|^2} = Z_x |K_i|^2 . (2.70)$$

From (2.70), and with reference to Table 2.3 and Figure 2.14, the secondary leakage inductance referred to the primary side, L'_{LX} , is now derived for each vector group.

Dd vector group,

$$L'_{LX} = \bar{a}^2 \frac{L_{Lx}}{3} \quad . \tag{2.71}$$

Dy vector group,

$$L'_{LX} = \left(\frac{\bar{a}}{\sqrt{3}}\right)^2 L_{Lx}$$
$$= \bar{a}^2 \frac{L_{Lx}}{3} . \qquad (2.72)$$

Yd vector group,

$$L'_{LX} = (\bar{a}\sqrt{3})^2 \frac{L_{Lx}}{3} = \bar{a}^2 L_{Lx} .$$
 (2.73)

Vector Group	L_m	L_L	R
Dd	$\frac{L_{mX}}{3}$	$\frac{L_{LX}}{3} + \bar{a}^2 \frac{L_{Lx}}{3}$	$\frac{R_X}{3} + \bar{a}^2 \frac{R_x}{3}$
Dy	$\frac{L_{mX}}{3}$	$\frac{L_{LX}}{3} + \bar{a}^2 \frac{L_{Lx}}{3}$	$\frac{R_X}{3} + \bar{a}^2 \frac{R_x}{3}$
Yd	L_{mX}	$L_{LX} + \bar{a}^2 L_{Lx}$	$R_X + \bar{a}^2 R_x$
Yy	L_{mX}	$L_{LX} + \bar{a}^2 L_{Lx}$	$R_X + \bar{a}^2 R_x$

Table 2.4: Simplified per phase equivalent circuit elements



Figure 2.15: Simplified per phase equivalent circuit

Yy vector group,

$$L'_{LX} = \bar{a}^2 L_{Lx} \quad . \tag{2.74}$$

From Figure 2.14 and (2.71), (2.72), (2.73) and (2.74), Table 2.4 presents the combined primary, and secondary referred, leakage inductance for the respective vector groups. The table also includes the winding resistance and magnetising inductance. Note that the resistance relationship is identical to the corresponding leakage inductance. It is clear from Table 2.4 that the circuit elements are determined by the primary connection since the circuit elements for the Dd and Dy vector groups are identical, and likewise for the Yd and Yy vector groups. A simplified per phase equivalent circuit is shown in Figure 2.15.

2.10 Conclusion

This chapter provided a review of power transformer materials and construction. It discussed the lamination material used in core construction and the most common forms of core used in modern power transformer design. Transformer windings were then introduced with attention focused on high voltage disc and low voltage helix windings. The chapter then briefly discussed transformer insulation and cooling arrangements. The latter section of the chapter introduced basic modelling of transformers operating at their nameplate frequency. It began with a single phase ideal transformer and progressed to the derivation of the per phase equivalent circuits for the most common transformer vector groups. The per phase equivalent circuit derivations are used in later chapters.

Chapter 3

Complex Permeability

3.1 Introduction

The manufacture of a core for a modern transformer out of thin, inherently resistive, laminations with anisotropic properties results in a magnetic circuit with high permeability at mains frequency. This permeability is attenuated with frequency as a result of magnetic skin effect [101] caused by induced eddy currents. In this chapter we investigate the effective bandwidth of core permeability, with a particular focus on the FRA spectrum (typically <10MHz [109]). We also investigate the effect that variations in the FRA test injection voltage will have on the core permeability.

This chapter is structured in the following manner. The latest research that has been conducted in this area is discussed in Section 3.2. Section 3.3 provides theory and a derivation of the effective complex permeability relationship. Section 3.4 develops a physically representative model and an estimation algorithm based on the work from Section 3.3. This model is used in Section 3.5 to accurately fit FRA data and determine quantitative values. Section 3.5 also demonstrates the significance of the core's permeability at higher frequencies through comparative testing against an air cored winding. Concluding remarks are then given in Section 3.6.

3.2 Background

Research to date on geometry based power transformer models has typically neglected the influence of the core beyond 50kHz [117, 49, 91]. A recent paper by Abeywickrama [15] extended this frequency range and stated that the effective complex permeability of the transformer core was significant above 100kHz for FRA testing. A paper by Bjerkan et al. [26] measured the inductance-frequency relationship up to 600kHz. Work by Tavner [110] on the coupling of discharge currents in the laminated steel stator core of electrical machines, showed a complex permeability greater than unity at 10MHz for 0.35mm laminations. The 0.35mm lamination thickness used in Tavner's work is in the upper range of the lamination thickness values used in transformer manufacture. By conjecture, complex



Figure 3.1: One dimensional eddy current model.

permeability should be similarly applicable to transformers at even greater frequencies for the smaller lamination thicknesses.

3.3 Theory

The aim of this section is to derive a relationship for the effective complex relative permeability of a power transformer core.

3.3.1 One Dimensional Analysis of Eddy Currents

The laminations in the core of a transformer have a thickness which is significantly smaller than their width and length. Furthermore, eddy currents tend to flow close to the conductor surface [107, 12]. As such, eddy currents in a transformer lamination can be studied in an approximate manner by considering one dimensional current flow as shown in Figure 3.1.

Faraday's law for electromagnetic induction is given as,

$$\nabla \mathbf{x} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} , \qquad (3.1)$$

where \mathbf{E} is the electric field strength and \mathbf{B} is the magnetic flux density. Relating the magnetic field intensity \mathbf{H} to the magnetic flux density \mathbf{B} using permeability we have,

$$\nabla \mathbf{x} \mathbf{E} = -\mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} , \qquad (3.2)$$

where μ_0 is the permeability of free space and μ_r is relative permeability. Next we replace the electric field strength **E** with current density **J**,

$$\nabla \mathbf{x} \mathbf{J} = -\sigma \mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} , \qquad (3.3)$$

where σ represents conductivity. With reference to Figure 3.1, the one dimensional analysis is performed with the magnetic field applied in the z direction and the induced eddy currents flowing in the x direction. This simplifies (3.3) to,

$$\frac{\partial J_x}{\partial y} = \sigma \mu_0 \mu_r \frac{\partial H_z}{\partial t} . \tag{3.4}$$

Utilising Ampére's law and neglecting displacement current since we are dealing with moving charges,

$$\nabla \mathbf{x} \mathbf{H} = \mathbf{J} \,. \tag{3.5}$$

Once again utilising the one dimensional model, (3.5) simplifies to,

$$\frac{\partial H_z}{\partial y} = J_x \ . \tag{3.6}$$

Differentiating (3.6),

$$\frac{\partial^2 H_z}{\partial y^2} = \frac{\partial J_x}{\partial y} . \tag{3.7}$$

Substituting (3.4) into (3.7) yields the Diffusion Equation [107],

$$\frac{\partial^2 H_z}{\partial y^2} = \sigma \mu_0 \mu_r \frac{\partial H_z}{\partial t} . \tag{3.8}$$

This is an important relationship which equates the delay and attenuation in the changing magnetic field, inside the lamination, to the induced eddy currents. Assuming that the field is time harmonic [32], the Diffusion Equation can be written as,

$$\frac{\partial^2 H_z}{\partial y^2} = j\omega \sigma \mu_0 \mu_r H_z \qquad (3.9)$$
$$= \gamma^2 H_z ,$$

where γ is the propagation constant given by

$$\gamma = \sqrt{j\omega\sigma\mu_0\mu_r} \ . \tag{3.10}$$

Since we consider only one dimension, the subscript z denoting direction is removed from H in all further references. Solving for H we have,

$$H = \beta \cosh(\gamma y) \quad \text{where } \beta \text{ is a constant }. \tag{3.11}$$

With reference to Figure 3.1, the flux in a cross sectional slice $2b\Delta_x$ is given by,

$$\Phi = \int_{-b}^{b} \mu_{0}\mu_{r}H \ \Delta_{x}dy$$

=
$$\int_{-b}^{b} \mu_{0}\mu_{r}\Delta_{x}\beta \cosh\left(\gamma y\right) dy$$

=
$$\frac{2\mu_{0}\mu_{r}\Delta_{x}\beta \sinh\left(\gamma b\right)}{\gamma} .$$
 (3.12)

Solving (3.12) for β ,

$$\beta = \frac{\gamma \Phi}{2\mu_0 \mu_r \Delta_x \sinh\left(\gamma b\right)} \,. \tag{3.13}$$

Substituting (3.13) into (3.11),

$$H = \frac{\gamma \Phi \cosh\left(\gamma y\right)}{2\mu_0 \mu_r \Delta_x \sinh\left(\gamma b\right)} \,. \tag{3.14}$$

As the magnetic field intensity is not attenuated at the surface of the lamination (y = b),

$$\dot{H} = \frac{\gamma \Phi \cosh\left(\gamma b\right)}{2\mu_0 \mu_r \Delta_x \sinh\left(\gamma b\right)} , \qquad (3.15)$$

where \hat{H} represents the magnetic field intensity at the surface. The space average magnetic flux density \bar{B} in the z direction relates the flux through the cross sectional slice as,

$$\bar{B} = \frac{\Phi}{2b\Delta_x} \ . \tag{3.16}$$

The effective complex permeability of a lamination is the ratio of the space average flux density to the surface magnetic field intensity, i.e.

$$\mu_e = \frac{\bar{B}}{\dot{H}} = \frac{\mu_0 \mu_r}{\gamma b} \tanh(\gamma b) . \qquad (3.17)$$

3.3.2 Lamination Anisotropy and Core Stacking Factor

Typically, modern power transformers use cold rolled grain oriented silicon steel in their laminations. The grain orientation provides the laminations with anisotropic properties. Due to these anisotropic properties, the permeability of the lamination in the longitudinal (rolled) direction is significantly greater than the permeability in the transverse direction [11]. In addition, though the introduction of joints into a core will result in a reduction in the overall effective permeability, the mitered overlapping of joints found in modern transformer construction will keep this effect relatively small [11]. On this basis and in order to simplify modelling, it is proposed that the core be considered continuous with magnetic flux restricted to flow in the longitudinal direction within the laminations.

When modelling a transformer core, it is generally not practical to consider laminations on an individual basis. This then requires a lamination stacking factor k to be



Figure 3.2: Laminated transformer core; n laminations of 2b thickness with Δ stacking excess, observing a magnetic flux density **B**.

introduced such that the core can be considered as a solid block [71]. With reference to Figure 3.2, a transformer core can be considered to be comprised of n laminations with an overall core depth of $2bn + \Delta$, where in this case Δ represents the stacking excess. In one dimension, the stacking factor k may then be defined as,

$$k = \frac{2bn}{2bn + \Delta} . \tag{3.18}$$

From [71], the longitudinal effective complex permeability of the core is given by,

$$\mu_e^* = k(\mu_e - 1) + 1 \approx k\mu_e . \tag{3.19}$$

This approximation is appropriate since lamination stacking factors for silicon steel can have values in the range of 0.95-0.98 [53]. Since permeability is typically referred to in its relative form (i.e. divided by μ_0), from (3.17) and (3.19), the longitudinal effective complex relative permeability of a transformer core can be approximated by,

$$\mu_s \approx \frac{k\mu_r}{\gamma b} \tanh\left(\gamma b\right) \,. \tag{3.20}$$

For the sake of brevity, the term effective permeability is used interchangeably with the term effective complex relative permeability, for μ_s . In addition, the frequency dependence of the effective permeability is not explicitly denoted in equations.



Figure 3.3: (a) Typical B/H magnetization curve for 3.5% silicon steel (Data from [29]) (b) Relative permeability versus magnetic field intensity based on the B/H curve.

3.3.3 Low Field Strength Relative Permeability

Frequency Response Analysis involves the injection of sine waves or an impulse signal into the windings of the transformer. The amplitude of the injected voltage is generally orders of magnitude less than the nameplate operating voltage of the transformer. Examples of the maximum injection voltages used in practice are $1V_{RMS}$ for the Omicron FRAnalyzer, $2.2V_{RMS}$ for the HP89410A, $3.5V_{RMS}$ for the Doble M5100 and M5200 and $7V_{RMS}$ for the Doble M5400. These voltages, even at mains frequency, will result in an induced magnetic field well below the operating "knee" of the B-H hysteresis curve of the core material. As such, these signals are considered to produce low field conditions. Furthermore, as the test frequency is increased, the inductive impedance increases, and the influence of parasitic winding capacitance also becomes apparent. This results in a decrease in the test current, and hence the associated magnetic field intensity, further reducing the relative permeability. Figure 3.3 presents a typical B/H curve for 3.5% silicon steel and the resulting relative permeability versus magnetic field intensity. However, under low field conditions, the relative permeability (μ_r) approaches the value of *initial permeability* (μ_i), which is defined as being the relative permeability at zero field strength [63],

$$\mu_r \approx \mu_i = \frac{1}{\mu_0} \lim_{H \to 0} \left[\frac{B}{H} \right].$$
(3.21)



Figure 3.4: Real component of the calculated effective permeability at 1MHz versus the initial permeability for a lamination thickness of 0.23mm, 0.30mm and 0.35mm.

Electrical Steel Grade	μ_i
BSI A.253, Ferrosil 253, Losil 25	150
Magnesil	300 - 1100
BSI C.86, Ferrosil 86, Transil 86	550
BSI grade 62, Alphasil 62, Unisil 62	1000
Silectron, Hypersil, Transcor 3X	1500

Table 3.1: Initial permeability for several grades of electrical steel. Data from [63].

From (3.20) and (3.21) the effective permeability of the core can be expressed, for the low field condition associated with FRA, as

$$\mu_s = \mu'_s - j\mu''_s \approx \frac{k\mu_i}{\gamma b} \tanh\left(\gamma b\right), \qquad (3.22)$$

where $\mu_{s}^{'}$ represents the real and $\mu_{s}^{''}$ the imaginary components of μ_{s} .

3.3.4 Effective Complex Relative Permeability of a Power Transformer at High Frequency

The aim of this section is to demonstrate that the effective permeability of a typical power transformer core remains significant at 1MHz. This is accomplished by plotting the real component of the calculated effective permeability at 1MHz versus the initial permeability,

using (3.22). The results are presented in Figure 3.4. The initial permeability is based on the typical values given in Table 3.1. A conductivity of $\sigma = 2.1 \times 10^6$ S/m was used, which is representative of common electrical steel grades - Unisil, M-2 through M-6, JIS: 30P105.

For the three lamination thicknesses, the calculated minimum for the real component of the effective permeability is at least an order of magnitude greater than unity at 1MHz (Figure 3.4). This implies that the coupling between windings on different phases must be considered when modelling the transformer at high frequencies.

3.3.5 Relationship between Effective Permeability and Winding Impedance

For a winding of N turns, with a permeability of unity, the inductance can be defined [89] as,

$$L_0 = \frac{\mu_0 N^2 A_{CS}}{l_c}$$
(3.23)

where l_c is the core length and A_{CS} the core cross sectional area. The inductance of a winding and its associated magnetic losses can be represented as an impedance which incorporates the effective permeability [103],

$$Z = jwL' + R'' (3.24)$$

$$= jwL_0(\mu'_s - j\mu''_s), \qquad (3.25)$$

where L' represents the ferromagnetic inductance and R'' the magnetic loss resistance. From (3.24) and (3.25),

$$L' = \mu'_{s} L_{0} \tag{3.26}$$

$$R'' = w\mu''_s L_0 . (3.27)$$

The relationships in (3.26) and (3.27) provide a convenient method to model the complex frequency dependent relationship between the transformer windings and the laminated electrical steel core at frequencies below self resonance. At higher frequencies, capacitance will need to be taken into account.

3.4 Estimation of Effective Permeability

The aim of this section is to demonstrate in a practical manner, that the effective permeability of a transformer core is significant above 1MHz. The approach we take is to estimate the permeability by fitting a physically representative model, based on the theory in section 3.3, to FRA data. The model parameters are only estimated up to the self resonant frequency such that the inductive contribution dominates the frequency response, hence the influence of parasitic capacitance can be neglected.


Figure 3.5: Sub-self resonant inductor model with respect to an FRA winding test.

3.4.1 Sub-Self Resonant Inductor Model

As shown in Section 3.3.5, at frequencies below the self resonant frequency, a winding can be modeled as a series combination of its inductance and its magnetic loss resistance. It is assumed that the winding's DC resistance and eddy current losses, due to skin and proximity effects, will remain small relative to the losses observed in the core, and hence can be neglected. Supporting arguments for this assumption are given in Appendix B. At higher frequencies as the permeability decays, the influence of flux leakage on the frequency response will increase. To improve the accuracy of the model, an element representing leakage inductance will complement the inductive contribution to account for quantitative variation between the winding and core cross sectional areas. The subself resonant inductor model is shown in Figure 3.5 where R'' represents the frequency dependent magnetic loss resistance, L' the ferromagnetic inductance and L_L the leakage inductance. Now,

$$v_{OUT}(w) = \frac{R}{R + R'' + jw(L' + L_L)} v_{IN}(w) , \qquad (3.28)$$

where R represents the 50 Ω FRA termination resistor. The transfer function, with reference to (3.22) and (3.25), becomes

$$G(jw) = \frac{v_{OUT}(w)}{v_{IN}(w)} = \frac{R}{R + jw(L_L + \mu_s L_0)}, \qquad (3.29)$$

where

$$L_0 = \frac{\mu_0 N^2 A_{CS}}{l_c} , \qquad (3.30)$$

$$L_L \approx \frac{\mu_0 N^2 (A_w - A_{CS})}{l_w} ,$$
 (3.31)

and N = number of winding turns,

 $A_{CS} = \text{cross sectional area of laminated core},$ $A_w = \text{cross sectional area of winding},$ $l_c = \text{length of laminated core},$ $l_w = \text{length of winding}.$

3.4.2 Estimation Algorithm

To estimate the effective permeability with respect to frequency, a constrained nonlinear optimisation algorithm is implemented utilising numerical computing software. This algorithm determines the best fit between the proposed model and the FRA data by finding the parameters that result in the lowest cost. The minimum cost, J, is calculated using the cumulative residual between corresponding model and data frequency points,

$$J = \left\| \log_{10} \left(\frac{\hat{G}(jw)}{H(jw)} \right) \right\|^2 , \qquad (3.32)$$

where H(jw) is the observed frequency response and $\hat{G}(jw)$ is the estimated transfer function for the model. Using a priori knowledge of R, L_0 and L_L , an estimate for the initial permeability, μ_i , and conductivity, σ , can be determined by,

$$[\hat{\mu}_i, \hat{\sigma}] = \arg\min_{\mu_i, \sigma} \{J\} , \qquad (3.33)$$

where $\hat{\mu}_i$ represents the estimated initial permeability and $\hat{\sigma}$, the estimated conductivity. Utilising the estimated and given parameters, an estimate of the effective permeability as a function of frequency can be obtained.

3.5 Experimental Verification

This section experimentally verifies the results developed in the previous sections. In particular, we demonstrate that:

- 1. Under FRA test conditions, the relative permeability approaches the initial permeability of the lamination material.
- 2. The effective permeability is significant at frequencies above 1MHz.
- 3. The effective permeability is greater than unity at frequencies exceeding 15MHz.

3.5.1 Wide-band Frequency Response Analysis Test Bed

A test bed is used to investigate the effective permeability of a power transformer core with respect to frequency. It is important that the test bed is representative of a power transformer under similar test conditions. Recall that the permeability of a transformer



Figure 3.6: Frequency response analysis test bed.



Figure 3.7: FRA test of the experimental test bed utilizing the single layer 200 turn laminated core inductor.

core is governed by the properties of the lamination material and the level of excitation. To ensure practical relevance to power transformer FRA, 0.35mm laminations (the upper bound for a typical transformer lamination [71]) are utilised as the core material for the test bed. The level of excitation induced by a typical transformer FRA test is many orders of magnitude less than at normal operating conditions. As such, the level of excitation applied to the test bed is of a similar magnitude to that of a typical power transformer during an FRA test.

One of the primary goals of the test bed is to constrain its resonant behaviour to



Figure 3.8: FRA test of the experimental test bed utilizing the single layer 5 turn laminated core inductor.

frequencies above those of interest. This is accomplished through the use of a single layer winding. The single layer winding possesses no interlayer capacitance and the interturn capacitance is significantly smaller than the capacitance to ground. As such, it does not possess the low frequency self resonance feature of multilayer windings. If the distributed winding to ground and measurement cable shunt capacitances are kept to a minimum, it is possible to obtain a frequency response that is dominated by the inductive components for frequencies greater than 1MHz. Figure 3.6 depicts the test bed used for all experiments in this chapter.

The test bed has two configurations. The first is for frequencies less than 1MHz and the second is for frequencies equal to or above 1MHz. The configuration details are given in Table 3.2.

Figure 3.7 shows the veracity of this experimental approach using the 200 turn, 0.35mm laminated core inductor. It is readily apparent that the resonance for this winding configuration occurs at frequencies greater than 1MHz. Hence, this particular configuration is only utilised for frequencies less than 1MHz. The resonant behaviour observed in Figure 3.7 is due to the interaction between the distributed inductance and the shunt capacitances to ground of both the winding and the measurement cables.

Figure 3.8 shows the frequency response for the 5 turn, 0.35mm laminated core inductor. It has a self resonant frequency at 30MHz. The resonant frequency is primarily influenced by the capacitance between turns, not to ground as in the previous configuration. This is due to the low number of turns on the winding. It is apparent from Figure 3.8 that this configuration is suitable for frequencies up to 15MHz without being unduly

3.5. EXPERIMENTAL VERIFICATION

	Frequencies < 1 MHz	Frequencies $\geq 1 \text{MHz}$			
Instrument	HP89410A Vector Analyser	E5071B Network Analyser			
Test cable/s	$Z_0 = 50\Omega, 100 \mathrm{pF/m}$	$Z_0=50\Omega,79.7\mathrm{pF/m}$			
Number of turns	200	5			
Winding length	140mm	4mm			
Wire gauge	22 B&S	24 B&S			
Winding diameter	90mm				
Laminated core length	900mm				
Lamination thickness	0.35mm				
Core dimension	$32 \times 37 \mathrm{mm}$				

Table 3.2: Test bed configuration details.



Figure 3.9: FRA test of the single layer 200 turn laminated core inductor using several test voltages $(0.5V_{RMS}, 1.0V_{RMS}, 1.5V_{RMS}, 2.0V_{RMS})$.

affected by the interturn capacitance.

An inductive response over a large range of frequencies provides an ideal platform to observe the effective permeability. This approach facilitates direct comparisons between a core constructed with laminations, typical of a power transformer, and that of an air cored equivalent, across a large range of frequencies.



Figure 3.10: Zoomed view of the frequency response of the single layer 200 turn laminated core inductor using several test voltages $(0.5V_{RMS}, 1.0V_{RMS}, 1.5V_{RMS}, 2.0V_{RMS})$.

3.5.2 Practical Confirmation of the Low Field Assumption

The relative permeability of a transformer core is governed by its B-H characteristic. Since the magnetic field intensity is a function of current, an injection voltage of constant amplitude results in a change of relative permeability with frequency due to the inductive impedance of the winding increasing. In Section 3.3.3, the conjecture was made that, due to the small injection voltages used by FRA test equipment relative to the nameplate voltage, the generated magnetic field intensity could be considered as a low field condition. That is $H \rightarrow 0$, over the FRA spectrum. As such, an assumption is made that the relative permeability approximates that of the *initial permeability* and remains relatively constant across the test frequency range. The validation of this assumption is important from a modelling perspective as it simplifies the complex non-linear relationship between an applied test voltage and the resulting relative permeability.

To confirm the low field conjecture in a practical setting, it is necessary to show that the frequency dependent winding inductance demonstrates a level of independence with respect to the FRA injection voltage. This is indicative of a constant relative permeability as per equations (3.21) and (3.22).

Experiments were conducted with a HP89410A vector analyzer using a range of test voltages from $0.5V_{RMS}$ to $2.0V_{RMS}$. These voltages are representative of those used by FRA testing authorities in the field. Furthermore, it is important to ensure that the test bed core is in a demagnetised condition prior to conducting the test. It has been shown [12] that the magnetic viscosity of electrical steel will reach its steady state condition after a period of approximately 24 hours. To ensure the validity of the test results, the

core was left in a de-energised state for a period greater than 24 hours. Experiments were then conducted in a sequence from lowest to highest injection voltages to ensure that the resulting core magnetisation level related only to the current test, and was not due to residual effects.

The experimental results are presented in Figure 3.9. Only very subtle differences in the frequency response can be observed, hence demonstrating independence with respect to FRA injection voltages. This is highlighted in Figure 3.10 where the frequency range has been restricted to show the small differences in magnitude for each injection voltage in more detail. This supports the assumption that, under FRA testing conditions, relative permeability will approximate the initial permeability.

Small errors that may be introduced due to this approximation can be minimised by conducting FRA tests using low amplitude injection voltages. However, there is a trade off between the low field response and the measurement device sensitivity i.e. acceptable signal to noise ratio of the frequency response.

3.5.3 Practical Demonstration of High Frequency Effective Permeability

To confirm that the effective permeability is still significant at 1MHz, an FRA test was conducted on the 200 turn single layer inductor containing a core constructed from 0.35mm silicon steel laminations. To provide a permeability reference of unity, an identical test was conducted on the winding in an air cored configuration. To complete the experiment, a test on the winding with a solid steel core was conducted to demonstrate the effective permeability tending to unity. The results are presented in Figure 3.11.

In Figure 3.11, it can be observed across all frequencies that the level of attenuation for the laminated core inductor is significantly greater than that of the reference air core inductor, despite having an identical winding. This clearly indicates that the effects of the magnetic core are the dominating factor in the frequency response. The frequency responses shown in Figure 3.11 support the theoretical results developed in Section 3.3.4. The laminated core inductor configuration has significantly more attenuation than the air cored version at 1MHz. This is indicative of a larger inductance which, since the same winding is used for all tests, corresponds to a permeability greater than unity.

The inductor with the solid steel core demonstrates the impact of magnetic skin effect. After only a few kilohertz, the skin depth ($\delta = \sqrt{2/w\sigma\mu_0\mu_r}$) is significantly smaller than the core dimensions, hence the effective permeability tends to unity and essentially follows the air cored winding frequency response as expected.

3.5.4 Estimation Algorithm for the Effective Permeability

In Section 3.4 an estimation algorithm was proposed in order to fit a sub-self resonant inductor model to FRA data. Using the test configuration shown in Figure 3.6, with test frequencies not exceeding 1MHz, FRA data was recorded. To accommodate material



Figure 3.11: FRA test of the 200 turn single layer inductor with an air core, solid steel core and a laminated electrical steel core (0.35mm laminations).

dependent variation in the conductivity for silicon steel (M235-35: $\sigma = 1.69 \times 10^6$ S/m, Ferrosil 86: $\sigma = 1.82 \times 10^6$ S/m, JIS 30P105: $\sigma = 2.1 \times 10^6$ S/m), a constraint was placed on the conductivity parameter in the parameter estimation algorithm. The lower and upper constraints were set to $\sigma_L = 1.5 \times 10^6$ S/m and $\sigma_H = 2.1 \times 10^6$ S/m respectively. The stacking factor was assumed to be k = 0.95. In conjunction with the test bed data specified in Table 3.2, the initial permeability and conductivity are estimated by the minimisation of the accumulated residual error between the frequency response of the model and the FRA data.

From (3.23) and with reference to Table 3.2,

$$L_0 = \frac{\mu_0 N^2 A_{CS}}{l_c} = 66 \times 10^{-6} \text{ H}.$$
 (3.34)

By considering the leakage inductance to be proportional to the difference in cross sectional area between the winding and the core, we can approximate the leakage inductance to be,

$$L_L \approx \frac{\mu_0 N^2 (A_w - A_{CS})}{l_w} = 1.9 \times 10^{-3} \text{ H},$$
 (3.35)

where A_w is the winding cross sectional area. From (3.10) and (3.21) the propagation constant can be found in terms of the initial permeability,

$$\gamma \approx \sqrt{j\omega\sigma\mu_0\mu_i}$$
. (3.36)

Substituting (3.36) into (3.22), noting that lamination thickness is equal to 2b,

$$\mu_s \approx \frac{k\mu_i}{\gamma b} \tanh\left(\gamma b\right).$$
(3.37)

Finally substituting (3.34), (3.35) and (3.37) into (3.29), the winding model transfer function is,

$$\hat{G}(jw) = \frac{50}{50 + jw \left[0.0019 + 0.36 \frac{\mu_i}{\gamma} \tanh\left(1.8 \times 10^{-4} \times \gamma\right) \right]} \,. \tag{3.38}$$

The transfer function (3.38) is utilised in the algorithm as described in Section 3.4.2 using the cost function (3.33). The estimated values for μ_i and σ are 835 and 1.72×10^6 S/m respectively. The resulting frequency response of the model (3.38) and the recorded FRA data is presented in Figure 3.12. As observed in this plot, the estimated model response is quite accurate in both magnitude and phase for four decades of frequency. This degree of accuracy supports the modelling approach and provides the necessary confidence in the estimated effective permeability (refer to Figure 3.13). From Figure 3.13, the value of the effective permeability at 1MHz is $\mu_s = 33 - 33j$. Based on this estimate, it can be concluded that there exists an effective permeability over thirty times greater than unity at 1MHz for a lamination thickness of 0.35mm. As previously discussed, this lamination thickness is typically the largest used in power transformer manufacture [71], hence even larger permeabilities would be obtained for smaller lamination thicknesses.

It should be noted that the effective permeability observed at high frequency is not large in comparison with the permeabilities observed under normal operating conditions at mains frequency. However, these values are significant when considering the subtleties involved in frequency response analysis and the associated coupling between windings of a three phase transformer.

3.5.5 Achieving an Effective Permeability of Unity

At frequencies greater than several megahertz, the measurement equipment itself becomes influential in the FRA test. The measurement cables, potentially several metres in length, can have a distributed capacitance of 100pF/m. In addition, the dimensions of the transformer windings under test are such that transmission line effects will not be negligible. As testing frequencies increase above a couple of megahertz the usefulness of FRA testing becomes questionable and it becomes more difficult to accurately record and assess. The recommended diagnostic range for a Doble M5400 FRA test device is 20Hz - 2MHz [3]. However, in order for this investigation to be rigorous, experiments were conducted to locate the frequency at which the effective permeability of a silicon steel core with 0.35mm laminations approaches unity.

To achieve this without self resonant interference and to minimise transmission line



Figure 3.12: Sub-self resonant inductor model fitting to FRA data of the 200 turn single layer inductor. Test voltage of $1.5V_{RMS}$.



Figure 3.13: Estimated effective permeability based on the sub-self resonant inductor model fitting; u' (real component) and u'' (imaginary component).

effects, the test bed with five single layer turns was used. The test was conducted using an Agilent E5071B Network Analyser in the same configuration as shown in Figure 3.6. The use of the network analyser facilitates a testing platform capable of taking into account the measurement equipment and associated high frequency transmission line effects. It does



Figure 3.14: High frequency FRA tests on the 5 turn single layer inductor with an air core and a laminated electrical steel core.

this by characterising the winding under test in terms of incident and reflected waves, and determining the associated scattering parameters [74]. An overview of scattering parameters is provided in Appendix C.

An experiment was conducted to determine the forward voltage gain scattering parameter [74] which represents the high frequency transfer function. The high frequency response for the test winding is shown in Figure 3.14.

The results in Figure 3.14 quantitatively demonstrate that the laminated core winding has a larger inductive impedance than the corresponding air cored winding, at frequencies exceeding 15MHz. As all the other parameters are equal, the effective permeability must also be greater than unity at frequencies larger than 15MHz. It should be noted that the frequency where the effective permeability reaches unity would be even higher for the smaller lamination thicknesses (<0.35mm) used in power transformer manufacture.

3.6 Conclusion

In this Chapter it has been shown that an FRA test on a power transformer should be considered a low field condition where the relative permeability approaches the initial permeability. The relative permeability remains approximately constant across the frequency spectrum for a fixed amplitude injection voltage despite the highly non-linear nature of the hysteresis characteristic. This results in a degree of independence with respect to the applied FRA test voltage. The research validated this assumption by performing FRA tests on a winding using a range of injection voltages. The results showed only very small differences between the frequency response at each voltage. The Chapter also showed that the effective complex relative permeability for a power transformer remains significant for frequencies exceeding 1MHz. This was demonstrated by comparing the level of inductive attenuation at 1MHz for a winding with and without the laminated core. A model was also fitted to the frequency response of the winding to confirm the theoretical conjectures. Finally, for experimental rigor, the frequency response of a winding using the 0.35mm laminations, was compared to an air cored version to demonstrate that an effective complex relative permeability of unity occurred at a frequency greater than 15MHz.

Chapter 4

Self and Mutual Inductance

4.1 Introduction

This chapter derives relationships for the self and mutual inductance between each of the windings of a three phase, double wound, core type transformer. Each of the relationships are in terms of the physical dimensions of the transformer core, the number of winding turns, effective permeability and leakage inductance. These relationships are integrated into the transformer model developed in Chapter 6.

This chapter is structured in the following manner. Section 4.2 discusses the different approaches used to model the mutual coupling between windings. Section 4.3 introduces the equivalent magnetic circuit and associated relationships for a three limb core type transformer. Section 4.4 derives the self inductance relationships of each winding. Section 4.5 derives relationships for the mutual inductance between each winding. Inductive disparity is discussed briefly in Section 4.6 and a transformer inductance matrix is formulated in Section 4.7. Concluding remarks are then given in Section 4.8.

4.2 Background

At its nameplate frequency, a power transformer performs close to the ideal, with measured losses in the order of 0.5% with respect to its power rating, and leakage flux losses in the vicinity of 5% relative to the mutual flux [24]. However at higher frequencies, for example during FRA testing, the permeability of the core will reduce and this will have the effect of increasing the influence of the flux leakage path. It is therefore important that flux leakage is also incorporated into a wide-band frequency transformer model.

There are several different methods used to model the mutual coupling of windings. The coupling coefficient model is used in simple two winding applications. This technique utilises a coefficient of coupling factor between zero and one to represent the degree of coupling between each of the windings. However, this approach does not model the inherent geometry of a transformer's magnetic circuit. Furthermore, it is not possible to use the coupling coefficient approach to model more than two coupled windings [44].



Figure 4.1: Magnetic equivalent circuit of a three phase two winding core type transformer

The most common approach taken for transformer modelling is known as the reluctance model. This modelling approach is physically representational, utilising a magnetic circuit based on the geometry of the transformer core. In the magnetic circuit, each winding is replaced with a magnetomotive source and each flux path, including those representative of leakage, with a corresponding reluctance. It is common practice in transient studies to utilise the principle of duality that exists between the electric and magnetic circuits [33, 86], i.e. to convert the magnetic circuit to its electric dual [75, 34]. Since this research is primarily concerned with the distributed interaction of parameters, the approach taken here is to use a combination of the magnetic circuit and its electric circuit dual.

The relationships derived in this chapter are based on a core type transformer with concentric windings. The leakage inductance is assumed to be restricted to the axial path between the high and low voltage windings (to be discussed in Chapter 5). The model is based on a single phase injection test, e.g. FRA, and therefore neglects the influence of zero sequence elements. Furthermore it is assumed that there is no indirect coupling between windings due to leakage flux.

4.3 Magnetic Circuit

The magnetic circuit proposed for the purpose of understanding the self and mutual inductance is shown in Figure 4.1. In this figure, \mathcal{F}_{A} represents the magnetomotive force (mmf) due to the high voltage winding of phase A, \mathcal{F}_{a} the mmf due to the low voltage

winding on phase A and so forth for the other phases. $\mathcal{R}_{\rm E}$ is the core limb reluctance, $\mathcal{R}_{\rm Y}$ is the core yoke reluctance and $\mathcal{R}_{\rm L}$ is the winding leakage flux reluctance. The dimensions of the core are l_E for the mean limb length and l_Y for the mean yoke length.

With reference to (2.1), the magnetomotive forces for the high and low voltage windings are,

$$\mathcal{F}_{A} = N_{X} i_{A} \tag{4.1}$$

$$\mathcal{F}_{\rm B} = N_X i_B \tag{4.2}$$

$$\mathcal{F}_{\rm C} = N_X i_C \tag{4.3}$$

$$\mathcal{F}_{\mathbf{a}} = N_x i_{\mathbf{a}} \tag{4.4}$$

$$\mathcal{F}_{\mathbf{b}} = N_x i_{\mathbf{b}} \tag{4.5}$$

$$\mathcal{F}_{\rm c} = N_x i_{\rm c} \quad , \tag{4.6}$$

where N_X and N_x represent the number of turns on the high and low voltage windings respectively, i_A the current in the HV winding phase A, i_a the current in the LV winding phase A, and similarly for the B and C phases.

The reluctance of the core can be defined in terms of the mean path length l, core cross sectional area A_{CS} , and the core permeability μ such that,

$$\mathcal{R} = \frac{l}{\mu A_{CS}} \quad . \tag{4.7}$$

For a three limb core transformer, the cross sectional area of the core can be assumed to be the same across all core sections [53]. In addition, the core permeability is a frequency dependent relationship that can be assumed to be approximately uniform throughout the core (Chapter 3 (3.22)). With reference to (4.7), it is observed that the reluctance is dependent upon the dimensions and layout of the physical magnetic circuit, and hence, $\mathcal{R}_{\rm E}$ and $\mathcal{R}_{\rm Y}$ are directly proportional to the limb and yoke length,

$$\mathcal{R}_E = \left[\frac{1}{\mu A_{CS}}\right] l_E , \qquad (4.8)$$

$$\mathcal{R}_{Y} = \left[\frac{1}{\mu A_{CS}}\right] l_{Y} . \tag{4.9}$$

The leakage reluctance, \mathcal{R}_L , is discussed in Section 5.4.

4.4 Winding Self Inductance

In this section we determine the self inductance of the windings relative to the core linear dimensions, cross sectional area and permeability. It is noted from the magnetic circuit model of Figure 4.1, that the windings on the two outside limbs, phases A and C, share common symmetry. As such, throughout this Chapter, windings on phases A and C with

equal turns are assumed equivalent. On this basis, derivations are only required for the A and B phase relationships.

4.4.1 Self Inductance

Self inductance is defined as [89],

$$L_X = \frac{\text{Flux linking winding X due to } i_X}{i_X} , \qquad (4.10)$$

where L_X is the self inductance of winding X and i_X is the associated winding current. This can be rewritten, taking into account flux linkage due to N_X turns as,

$$L_X = \frac{N_X \Phi_X}{i_X} \quad , \tag{4.11}$$

where the winding observes a flux of Φ_X . From the definition of magnetomotive force [89],

$$\mathcal{F}_{\mathcal{X}} = N_{\mathcal{X}} i_{\mathcal{X}} = \mathcal{R}_{\mathcal{X}} \Phi_{\mathcal{X}} \tag{4.12}$$

Substituting (4.12) into (4.11),

$$L_X = \frac{N_X^2}{\mathcal{R}_X} \quad . \tag{4.13}$$

From (4.13), it is noted that the self inductance is dependent upon the reluctance observed by the winding X.

4.4.2 Self Inductance of the Outside Limb Winding

Using superposition to determine the observed reluctance of a particular winding, the mmf sources of all of the other windings are short circuited. Figure 4.2(a) shows the magnetic circuit for the reluctance of a winding wound on the outside limbs, i.e. phases A and C. This circuit consists of a winding leakage path $\mathcal{R}_{\rm L}$, and the magnetic path around the core consisting of a combination of limb and yoke reluctances, \mathcal{R}_E and \mathcal{R}_Y . By simplifying the magnetic core path to a single value $\mathcal{R}_{\rm MA}$, the resulting reluctance of the winding can be considered as a magnetic core and leakage parallel path as per Figure 4.2(b),

$$\mathcal{R}_{A} = \mathcal{R}_{MA} \| \mathcal{R}_{L}$$

$$\therefore \mathcal{R}_{A} = \frac{\mathcal{R}_{MA} \mathcal{R}_{L}}{\mathcal{R}_{MA} + \mathcal{R}_{L}} ,$$
 (4.14)



Figure 4.2: Magnetic circuit transformation of a winding on the core's outside limb. (a) Magnetic circuit based on core geometry. (b) Simplified magnetic circuit. (c) Duality based equivalent electrical circuit.

where \mathcal{R}_{A} is the reluctance observed by a winding on an outside limb. This result is consistent for both the high and low voltage windings. From (4.8) and (4.9),

$$\mathcal{R}_{\mathrm{MA}} = \left[\frac{1}{\mu A_{CS}}\right] \left[l_E + l_Y + l_E \| (l_E + l_Y)\right]$$
$$= \left[\frac{1}{\mu A_{CS}}\right] \left[l_E + l_Y + \left(\frac{l_E (l_E + l_Y)}{2l_E + l_Y}\right)\right]$$
$$\therefore \mathcal{R}_{\mathrm{MA}} = \left[\frac{1}{\mu A_{CS}}\right] \left[\frac{(l_E + l_Y) (3l_E + l_Y)}{2l_E + l_Y}\right] . \tag{4.15}$$

From (4.13) and (4.15), the core component of self inductance for the high voltage winding on the outside limb, L_{MAA} , can now be determined,

$$L_{MAA} = \frac{N_X^2}{\mathcal{R}_{MA}}$$

$$= \frac{N_X^2}{\left[\frac{1}{\mu A_{CS}}\right] \left[\frac{(l_E+l_Y)(3l_E+l_Y)}{2l_E+l_Y}\right]}$$

$$\therefore L_{MAA} = \frac{\mu A_{CS} N_X^2 \left(2l_E+l_Y\right)}{(l_E+l_Y) \left(3l_E+l_Y\right)} . \qquad (4.16)$$

Each of the high voltage windings can be assumed to have the same number of turns, the generic term N_X is employed. Similarly, for the core component of self inductance for the low voltage outside limb winding, employing N_x for the number of turns on the low voltage winding,

$$L_{Maa} = \frac{\mu A_{CS} N_x^2 \left(2l_E + l_Y\right)}{\left(l_E + l_Y\right) \left(3l_E + l_Y\right)} . \tag{4.17}$$

Taking into account the transformer's turns ratio (2.5), (4.17) can then be simplified to,

$$L_{Maa} = \frac{L_{MAA}}{\bar{a}^2} . aga{4.18}$$

The leakage reluctance is assumed equivalent for both the high and low voltage windings¹, hence from (4.13),

$$L_{LA} = \frac{N_X^2}{\mathcal{R}_L} \tag{4.19}$$

$$L_{La} = \frac{N_x^2}{\mathcal{R}_L} \tag{4.20}$$

where L_{LA} is the high voltage outside limb winding leakage inductance, and L_{La} is the corresponding low voltage outside limb winding leakage inductance. Hence from (4.19) and (4.20), a relationship for the leakage reluctance in terms of the corresponding leakage

¹This assumption is justified in Section 5.4

inductance and winding turns is,

$$\mathcal{R}_L = \frac{N_X^2}{L_{LA}} = \frac{N_x^2}{L_{La}} \ . \tag{4.21}$$

From (4.13), (4.14) and (4.19), the self inductance of the high voltage outside limb winding, L_{AA} , is given by,

$$L_{AA} = \frac{N_X^2}{\mathcal{R}_A}$$

= $N_X^2 \frac{\mathcal{R}_{MA} + \mathcal{R}_L}{\mathcal{R}_{MA} \mathcal{R}_L}$
= $\frac{N_X^2}{\mathcal{R}_L} + \frac{N_X^2}{\mathcal{R}_{MA}}$
 $L_{AA} = L_{LA} + L_{MAA}$. (4.22)

Similarly, for the self inductance of the low voltage outside limb winding, L_{aa} ,

· .

$$L_{aa} = L_{La} + L_{Maa} \quad . \tag{4.23}$$

This result indicates that the self inductance of a winding on the outside limb can be considered as the series combination of the winding's leakage and magnetic core inductance.

With reference to Figure 4.2(b), the same result can also be achieved through the implementation of the principle of duality [33]. The electrical dual is found by replacing the magnetic mesh circuits with their nodal electrical equivalents. This is accomplished by placing a reference node inside each magnetic circuit loop, (i) and (j), and an additional reference node outside the circuit, (k). Lines are then drawn between any two nodes of the magnetic circuit that pass through a single circuit element. The electrical equivalent joins the reference nodes and replaces each element with its dual; reluctance with inductance, mmf with current. The electrical dual of the magnetic circuit is shown in Figure 4.2(c) indicating the series combination of the leakage and magnetic core inductance as per (4.22) and (4.23).

4.4.3 Self Inductance of the Centre Limb Winding

In a similar manner, the self inductance of the centre limb of the transformer core can be derived. For the reluctance of the core, from (4.8) and (4.9) and with reference to Figure 4.3(a),

$$\mathcal{R}_{\text{MB}} = \left[\frac{1}{\mu A_{CS}}\right] \left[l_E + (l_E + l_Y) \parallel (l_E + l_Y)\right]$$
$$= \left[\frac{1}{\mu A_{CS}}\right] \left[\frac{3l_E + l_Y}{2}\right]$$
(4.24)

From (4.13) and (4.24), the core component of self inductance for the high voltage centre



Figure 4.3: Magnetic circuit transformation of a winding on the core's centre limb. (a) Magnetic circuit based on core geometry. (b) Simplified magnetic circuit. (c) Duality based equivalent electrical circuit.

limb winding is given by,

$$L_{MBB} = \frac{N_X^2}{\mathcal{R}_{MB}}$$
$$= \frac{N_X^2}{\left[\frac{1}{\mu A_{CS}}\right] \left[\frac{3l_E + l_Y}{2}\right]}$$
$$\therefore \ L_{MBB} = \frac{2\mu A_{CS} N_X^2}{3l_E + l_Y} . \tag{4.25}$$

Similarly, for the low voltage centre limb winding, the core's contribution to the self inductance is,

$$L_{Mbb} = \frac{2\mu A_{CS} N_x^2}{3l_E + l_Y} \quad . \tag{4.26}$$

Taking into account the transformer turns ratio, (4.26) can be rewritten as,

$$L_{Mbb} = \frac{L_{MBB}}{\bar{a}^2} \quad . \tag{4.27}$$

The centre winding's electrical dual, Figure 4.3(c), is once again just the series combination of the leakage inductance and the core's contribution to the respective self inductance. For the high voltage winding this is,

$$L_{BB} = L_{LB} + L_{MBB} \quad , \tag{4.28}$$

where L_{BB} is the self inductance of the high voltage centre limb winding and L_{LB} is the leakage inductance of the high voltage centre limb winding. Similarly, for the self inductance of the low voltage centre limb winding, L_{bb} ,

$$L_{bb} = L_{Lb} + L_{Mbb} , (4.29)$$

where L_{Lb} is the leakage inductance of the low voltage centre limb winding.

Self inductance relationships have now been derived for each of the associated windings of a core type transformer. The derivations take into account leakage loss and core path length variation.

4.5 Winding Mutual Inductance

In this section we derive mutual inductance relationships for both the HV and LV windings of the transformer. The derivations are divided into two categories, those sharing the same transformer core limb, which are classified as intraphase mutuals, and those on separate transformer core limbs, classified as interphase mutuals. These categories include both high and low voltage winding variants and the resulting derivations are in terms of the core linear dimensions, cross sectional area and permeability. As with self inductance, the windings of phase A and phase C are considered interchangeable due to the symmetrical nature of the core. On this basis, only derivations for phase A and B are considered.

The mutual inductance between two winding sections is defined as being the flux linking one winding due to the current in another. This can be expressed mathematically [89] as,

$$L_{XZ} = \frac{\text{Flux linking winding Z due to } i_X}{i_X} , \qquad (4.30)$$

where L_{XZ} is the mutual inductance between generic windings X and Z. This can be rewritten, taking into account the flux linkage of winding Z due to N_Z turns as,

$$L_{XZ} = \frac{N_Z \Phi_{XZ}}{i_X} \quad , \tag{4.31}$$

where Φ_{XZ} is the flux linking winding Z due to current in winding X. To determine the degree of mutual coupling between two windings it is necessary to determine the amount of flux received by a winding relative to the current generating the flux of the other.

4.5.1 Intraphase Mutual Inductance

For the intraphase mutual inductance, a derivation of the mutual inductance between the high and low voltage windings that share a common core limb is required. This category is subdivided into two areas, windings on the outside limbs and windings on the centre limb.

Outside Limb Windings

From (4.31), we consider the flux linking the low voltage winding of phase A due to current in the high voltage winding. With reference to the magnetic circuit model of Figure 4.4(a), the low voltage winding shares the same magnetic core flux as the high voltage winding, Φ_{MA} . From (2.17),

$$\Phi_{MA} = \frac{\mathcal{F}_A}{\mathcal{R}_{MA}} = \frac{N_X i_A}{\mathcal{R}_{MA}} \quad , \tag{4.32}$$

where \mathcal{F}_A is the high voltage winding magnetomotive force due to a current i_A , and \mathcal{R}_{MA} is the observed core reluctance defined in (4.15). From (4.15) and (4.32),

$$\Phi_{MA} = \frac{N_X i_A}{\left[\frac{1}{\mu A_{CS}}\right] \left[\frac{(l_E + l_Y)(3l_E + l_Y)}{2l_E + l_Y}\right]}$$

$$\therefore \Phi_{MA} = \frac{\mu A_{CS} N_X i_A (2l_E + l_Y)}{(l_E + l_Y) (3l_E + l_Y)} . \qquad (4.33)$$

From Faraday's law, the magnitude of an induced voltage on a winding due to a changing flux is defined as,

$$v_k = -N_k \frac{d\Phi_k}{dt} \quad , \tag{4.34}$$



Figure 4.4: Magnetic circuit for intra-phase mutual inductance. (a) High voltage phase A winding to low voltage phase A winding. (b) High voltage phase B winding to low voltage phase B winding.

where Φ_k is the flux through N_k turns of winding k, resulting in a voltage across the winding of v_k . Applying Faraday's law, (4.34), to (4.33),

$$v_{a} = -N_{x} \frac{d\Phi_{MA}}{dt}$$

= $-N_{x} \frac{d}{dt} \left[\frac{\mu A_{CS} N_{X} i_{A} (2l_{E} + l_{Y})}{(l_{E} + l_{Y}) (3l_{E} + l_{Y})} \right]$
= $-\frac{\mu A_{CS} N_{x} N_{X} (2l_{E} + l_{Y})}{(l_{E} + l_{Y}) (3l_{E} + l_{Y})} \frac{di_{A}}{dt}$, (4.35)

where v_a is the induced voltage on the phase A low voltage winding due to the flux generated by the phase A high voltage winding. Equation (4.34) can be rewritten as,

$$v_k = L_k \, \frac{di_k}{dt} \quad , \tag{4.36}$$

where L_k is the inductance of winding k. From (4.35), it can be deduced that,

$$L_{MAa} = \frac{\mu A_{CS} N_x N_X \left(2l_E + l_Y\right)}{\left(l_E + l_Y\right) \left(3l_E + l_Y\right)} \quad , \tag{4.37}$$

where L_{MAa} is the mutual inductance between the high and low voltage windings on the outside limb of a core type transformer. By extension, the same result is obtained from the perspective of the mutual inductance for the low to the high voltage winding,

$$L_{MaA} = L_{MAa} \quad . \tag{4.38}$$

These results can be equally applied to phase C.

Centre Limb Windings

With reference to the magnetic circuit model of Figure 4.4(b), the low voltage winding shares the same magnetic core flux as the high voltage winding, Φ_{MB} . From (2.17) and (4.24),

$$\Phi_{MB} = \frac{\mathcal{F}_B}{\mathcal{R}_{MB}}$$

$$= \frac{N_X i_B}{\left[\frac{1}{\mu A_{CS}}\right] \left[\frac{3l_E + l_Y}{2}\right]}$$

$$\therefore \Phi_{MB} = \frac{2\mu A_{CS} N_X i_B}{(3l_E + l_Y)} . \qquad (4.39)$$

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From Faraday's law, (4.34),

$$v_b = -N_x \frac{d\Phi_{MB}}{dt}$$

= $-\frac{2\mu A_{CS} N_x N_X}{(3l_E + l_Y)} \frac{di_B}{dt}$, (4.40)

where v_b is the induced voltage on the phase *B* low voltage winding due to the flux generated by the phase *B* high voltage winding. Noting the inductance of (4.36), the mutual inductance can be determined from (4.40) to be,

$$L_{MBb} = \frac{2\mu A_{CS} N_x N_X}{(3l_E + l_Y)} \quad , \tag{4.41}$$

where L_{MBb} is the mutual inductance between the high and low voltage windings on the centre limb of a core type transformer. By extension, as with the outside limb relationship, the mutual inductance from the low voltage winding to the high voltage winding, L_{MbB} , is the same as (4.41),

$$L_{MbB} = L_{MBb} \quad . \tag{4.42}$$

4.5.2 Interphase Mutual Inductance

For the interphase mutual inductance, a derivation of the mutual inductance between windings on different limbs is required. This category is also subdivided into two areas. The first area is the mutual inductance between windings on opposite outside limbs, i.e. between phases A and C. The second is the mutual inductance between an outside limb winding and a centre limb winding, e.g. phases A and B. Both categories consider all combinations of high and low voltage windings.

Outside Limb Windings

To determine the mutual inductance that exists between windings on opposing outside limbs, the high voltage windings of phases A and C are considered initially. The results are then extended to the various high and low voltage winding combinations of the two phases.

With reference to the magnetic circuit model of Figure 4.5(a), the magnetic core flux generated by the magnetomotive force \mathcal{F}_A , as derived in (4.33), is,

$$\Phi_{MA} = \frac{\mathcal{F}_A}{\mathcal{R}_{MA}} = \frac{\mu A_{CS} N_X i_A \left(2l_E + l_Y\right)}{\left(l_E + l_Y\right) \left(3l_E + l_Y\right)} \quad . \tag{4.43}$$

The proportion of the flux Φ_{MA} linking the high voltage winding of phase C, can be



(b)

Figure 4.5: Magnetic circuit for inter-phase mutual inductance. (a) High voltage phase A winding to high voltage phase C winding. (b) High voltage phase A winding to high voltage phase B winding.

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determined by flux division based on the associated path reluctance,

$$\Phi_{MAC} = \frac{\mathcal{R}_E}{\mathcal{R}_E + (\mathcal{R}_E + \mathcal{R}_Y)} \Phi_{MA}$$
$$= \frac{\mathcal{R}_E}{(2\mathcal{R}_E + \mathcal{R}_Y)} \Phi_{MA} . \qquad (4.44)$$

Since the core is considered to have a uniform cross sectional area and permeability, (4.44) can be rewritten in terms of core path length,

$$\Phi_{MAC} = \frac{l_E}{(2l_E + l_Y)} \Phi_{MA} \quad . \tag{4.45}$$

Substituting (4.43) into (4.45),

$$\Phi_{MAC} = \frac{l_E}{(2l_E + l_Y)} \frac{\mu A_{CS} N_X i_A (2l_E + l_Y)}{(l_E + l_Y) (3l_E + l_Y)}$$

$$\therefore \Phi_{MAC} = \frac{\mu A_{CS} N_X i_A l_E}{(l_E + l_Y) (3l_E + l_Y)} . \qquad (4.46)$$

From Faraday's law, (4.34), the voltage of the high voltage phase C winding based on a flux of (4.46) is,

$$v_C = -N_X \frac{d\Phi_{MAC}}{dt}$$

= $-N_X \frac{d}{dt} \left[\frac{\mu A_{CS} N_X i_A l_E}{(l_E + l_Y) (3l_E + l_Y)} \right]$
$$\therefore v_C = - \frac{\mu A_{CS} N_X^2 l_E}{(l_E + l_Y) (3l_E + l_Y)} \frac{di_A}{dt} , \qquad (4.47)$$

where v_C is the induced voltage across the high voltage phase C winding. We can now obtain the inductance component of (4.47),

$$L_{MAC} = L_{MCA} = \frac{\mu A_{CS} N_X^2 l_E}{(l_E + l_Y) (3l_E + l_Y)} , \qquad (4.48)$$

where L_{MAC} and L_{MCA} are the mutual inductances between the high voltage phase A and C windings. Note that $L_{MAC} = L_{MCA}$ due to the symmetry of phases A and C.

By extension, the results from (4.48) can be replicated for the low voltage winding by simply substituting the low voltage turns, N_x for the high voltage turns, N_X ,

$$L_{Mac} = L_{Mca} = \frac{\mu A_{CS} N_x^2 l_E}{(l_E + l_Y) (3l_E + l_Y)} , \qquad (4.49)$$

where L_{Mac} and L_{Mca} are the mutual inductances between the outside limb low voltage windings.

The interphase mutual inductance relationships between the high and low voltage

windings can be similarly derived. Applying Faraday's law to determine the induced voltage on the phase C low voltage winding due to the high voltage phase A winding generated flux, (4.34) and (4.46),

$$v_c = -N_x \frac{d\Phi_{MAC}}{dt}$$

$$\therefore v_c = -\frac{\mu A_{CS} N_x N_X l_E}{(l_E + l_Y) (3l_E + l_Y)} \frac{di_A}{dt} , \qquad (4.50)$$

where v_c is the induced voltage on the low voltage phase C winding. Hence, from (4.50), the mutual inductance between a high and low voltage winding on opposite outside limbs, is given by,

$$L_{MAc} = L_{MaC} = L_{MCa} = L_{McA} = \frac{\mu A_{CS} N_x N_X l_E}{(l_E + l_Y) (3l_E + l_Y)} .$$
(4.51)

Outside and Centre Limb Windings

•

In this section we determine the mutual inductance that exists between the outside and centre limb windings. The derivation initially focuses on the high voltage phase A and B windings, and then extends to the high and low voltage winding combinations of the these two phases.

As discussed in Section 4.5.2, the proportion of the flux Φ_{MA} linking a winding of another phase can be determined by flux division based on the associated core path length. With reference to Figure 4.5(b) and (4.43), for a winding on phase *B* the associated flux Φ_{MAB} is,

$$\Phi_{MAB} = \frac{l_E + l_Y}{l_E + (l_E + l_Y)} \Phi_{MA}$$

$$= \frac{l_E + l_Y}{(2l_E + l_Y)} \frac{\mu A_{CS} N_X i_A (2l_E + l_Y)}{(l_E + l_Y) (3l_E + l_Y)}$$

$$\therefore \Phi_{MAB} = \frac{\mu A_{CS} N_X i_A}{(3l_E + l_Y)} . \qquad (4.52)$$

Applying Faraday's law, (4.34), to the high voltage phase B winding based on a flux of (4.52),

$$v_B = -N_X \frac{d\Phi_{MAB}}{dt}$$

= $-\frac{\mu A_{CS} N_X^2}{(3l_E + l_Y)} \frac{di_A}{dt}$, (4.53)

where v_B is the induced voltage on the high voltage phase B winding. The inductance component of (4.53) is,

$$L_{MAB} = \frac{\mu A_{CS} N_X^2}{(3l_E + l_Y)} \quad , \tag{4.54}$$

4.6. INDUCTIVE DISPARITY

where L_{MAB} is the mutual inductance relationship from the high voltage phase A winding to the high voltage phase B winding.

The flux observed by the high voltage phase A winding relative to the mmf of a high voltage winding on phase B is given by,

$$\Phi_{MBA} = \frac{l_E + l_Y}{(l_E + l_Y) + (l_E + l_Y)} \cdot \Phi_{MB}$$
$$= \frac{1}{2} \frac{2\mu A_{CS} N_X i_B}{(3l_E + l_Y)}$$
$$\therefore \Phi_{MBA} = \frac{\mu A_{CS} N_X i_B}{(3l_E + l_Y)} . \qquad (4.55)$$

Since the respective flux relationships of (4.52) and (4.55) are equivalent based on $i_A = i_B$, then the mutual inductances can also be assumed to be equivalent,

$$L_{MAB} = L_{MBA} \tag{4.56}$$

where L_{MBA} is the mutual inductance from the high voltage phase B winding to the high voltage phase A winding.

Similarly, the results from (4.54) and (4.56) can be applied to the low voltage windings by substituting the low voltage turns, N_x for the high voltage turns, N_X ,

$$L_{Mab} = L_{Mba} = \frac{\mu A_{CS} N_x^2}{(3l_E + l_Y)}$$
(4.57)

where L_{Mab} and L_{Mba} are the mutual inductances between a low voltage outside limb winding and a low voltage centre limb winding. In addition, as in Section 4.5.2, the high/low voltage mutual inductances are,

$$L_{MAb} = L_{MaB} = L_{MBa} = L_{MbA} = \frac{\mu A_{CS} N_x N_X}{(3l_E + l_Y)} \quad . \tag{4.58}$$

As discussed in the introduction, all of the results derived in this section for phase A can be equally applied to phase C.

4.6 Inductive Disparity

Due to the intrinsic variation in the reluctance of windings on the outside limbs compared to their counterparts on the centre limb, there exists a disparity in the corresponding self inductance values. This is demonstrated by the high voltage self inductances of phases A(4.16), and B (4.25) in Section 4.4. By comparing these two inductances, the degree of inductive disparity can be quantified in terms of the core's limb and yoke dimensions,

$$\Gamma = \frac{L_{MAA}}{L_{MBB}}$$

$$= \frac{\mu A_{CS} N_X^2 (2l_E + l_Y)}{(l_E + l_Y) (3l_E + l_Y)} \frac{3l_E + l_Y}{2\mu A_{CS} N_X^2}$$

$$\therefore \Gamma = \frac{2l_E + l_Y}{2(l_E + l_Y)}$$
(4.59)

where Γ is a figure representing the level of inductive disparity between phases due to the core topology of the transformer. Since the reluctance path observed by the outside windings will always be greater than that observed by the centre windings, the defined inductive disparity of (4.59) will always be less than one. Based on the core length ratio constraints from [34], inductive disparity has a typical range of,

$$0.60 \le \Gamma \le 0.75$$
 . (4.60)

The effects of inductive disparity are discussed in Appendix A.

4.7 Transformer Inductance Matrix

The magnetic core based self and mutual inductances developed in Sections 4.4 and 4.5 can be simplified through use of the transformer turns ratio \bar{a} , inductive disparity Γ , and a dimension ratio constant Λ which is introduced in Section 4.7.2.

4.7.1 Magnetic Core Self Inductance Terms

The first step is to take the magnetic core's contribution to self inductance for the high voltage phase A winding, and assume it is equivalent to a generic inductance term \tilde{L} ,

$$L_{MAA} = \tilde{L} \quad , \tag{4.61}$$

where, from (4.16), \tilde{L} is,

$$\tilde{L} = \frac{\mu A_{CS} N_X^2 \left(2l_E + l_Y\right)}{\left(l_E + l_Y\right) \left(3l_E + l_Y\right)} \ . \tag{4.62}$$

On this basis, due to core symmetry, the same expression can also be directly assigned to the phase C high voltage winding magnetic core self inductance,

$$L_{MCC} = \tilde{L} \quad . \tag{4.63}$$

Similarly, via use of the inductive disparity defined in (4.59), the high voltage phase B winding magnetic core self inductance is,

$$L_{MBB} = \frac{\tilde{L}}{\Gamma} \quad . \tag{4.64}$$

The equivalent relationships for the low voltage winding magnetic core self inductance utilising the transformer turns ratio are,

$$L_{Maa} = \frac{\tilde{L}}{\bar{a}^2} \tag{4.65}$$

$$L_{Mbb} = \frac{L}{\Gamma \bar{a}^2} \tag{4.66}$$

$$L_{Mcc} = \frac{L}{\bar{a}^2} . aga{4.67}$$

The high/low voltage common limb winding mutual inductances can be rewritten as,

$$L_{MAa} = L_{MaA} = \frac{\tilde{L}}{\bar{a}}$$
(4.68)

$$L_{MBb} = L_{MbB} = \frac{L}{\prod_{\tilde{a}} \tilde{a}}$$
(4.69)

$$L_{MCc} = L_{McC} = \frac{L}{\bar{a}} . aga{4.70}$$

4.7.2 Magnetic Core Mutual Inductances

In a similar fashion to Section 4.7.1, this section simplifies the magnetic core mutual inductance derivations of Section 4.5.

Outside limb mutual inductance terms

For the outside limb mutual inductance terms, L_{AC} and L_{CA} , from (4.48) and (4.62),

$$L_{MAC} = L_{MCA} = \frac{\mu A_{CS} N_X^2 l_E}{(l_E + l_Y) (3l_E + l_Y)} \\ = \left[\frac{l_E}{2l_E + l_Y} \right] \tilde{L} .$$
(4.71)

Defining a dimension ratio constant Λ as,

$$\Lambda = \frac{2l_E + l_Y}{l_E} \quad , \tag{4.72}$$

allows (4.71) to be simplified,

$$L_{MAC} = L_{MCA} = \frac{\tilde{L}}{\Lambda} \quad . \tag{4.73}$$

From (4.73) for L_{ac} and L_{ca} , and considering the turns ratio,

$$L_{Mac} = L_{Mca} = \frac{\tilde{L}}{\Lambda \bar{a}^2} \quad . \tag{4.74}$$

The reduced relationship for the combination high/low voltage winding mutual inductance of the core's outside limbs can be defined as,

$$L_{MAc} = L_{MaC} = L_{MCa} = L_{McA} = \frac{L}{\Lambda \bar{a}} \quad . \tag{4.75}$$

Outside/centre limb mutual inductances

For the outside to centre limb mutual inductances, from (4.54) and (4.56),

$$L_{MAB} = \frac{\mu A_{CS} N_X^2}{(3l_E + l_Y)}$$
$$= \frac{1}{2} L_{MBB}$$
$$\therefore \ L_{MAB} = L_{MBA} = \frac{\tilde{L}}{2\Gamma} , \qquad (4.76)$$

and noting the symmetry of phases A and C,

$$L_{MBC} = L_{MCB} = \frac{\tilde{L}}{2\Gamma} \quad . \tag{4.77}$$

Hence for the low voltage windings,

$$L_{Mab} = L_{Mba} = \frac{\tilde{L}}{2\Gamma\bar{a}^2} \quad , \tag{4.78}$$

and,

$$L_{Mbc} = L_{Mcb} = \frac{\tilde{L}}{2\Gamma\bar{a}^2} \quad . \tag{4.79}$$

For the combination high/low voltage winding mutual inductance of the core's outside/centre limbs,

$$L_{MAb} = L_{MaB} = L_{MBa} = L_{MbA} = \frac{L}{2\Gamma\bar{a}} \quad , \tag{4.80}$$

and,

$$L_{MBc} = L_{MbC} = L_{MCb} = L_{McB} = \frac{L}{2\Gamma\bar{a}}$$
 (4.81)

4.7.3 Inductance Matrix

A complete inductance matrix that details the associated self and mutual inductances defined in this chapter is presented in Table 4.1. Each inductive element of Table 4.1 is derived in terms of a combination of the following parameters,

• Core permeability $[\mu]$

\mathbf{L}	A	В	C	a	b	с
A	$\tilde{L} + L_{LA}$	$\frac{\tilde{L}}{2\Gamma}$	$\frac{\tilde{L}}{\Lambda}$	$\frac{\tilde{L}}{\bar{a}}$	$rac{ ilde{L}}{2\Gammaar{a}}$	$\frac{\tilde{L}}{\Lambda \bar{a}}$
В	$rac{ ilde{L}}{2\Gamma}$	$\frac{\tilde{L}}{\Gamma} + L_{LB}$	$rac{ ilde{L}}{2\Gamma}$	$rac{ ilde{L}}{2\Gammaar{a}}$	$rac{ ilde{L}}{\Gamma ar{a}}$	$rac{ ilde{L}}{2\Gammaar{a}}$
C	$\frac{\tilde{L}}{\Lambda}$	$\frac{\tilde{L}}{2\Gamma}$	$\tilde{L} + L_{LC}$	$rac{ ilde{L}}{\Lambda ar{a}}$	$rac{ ilde{L}}{2\Gamma ar{a}}$	$\frac{\tilde{L}}{\bar{a}}$
a	$rac{ ilde{L}}{ar{a}}$	$rac{ ilde{L}}{2\Gammaar{a}}$	$rac{ ilde{L}}{\Lambda ar{a}}$	$\frac{\tilde{L}}{\bar{a}^2} + L_{La}$	$rac{ ilde{L}}{2\Gamma ar{a}^2}$	$\frac{\tilde{L}}{\Lambda \bar{a}^2}$
b	$rac{ ilde{L}}{2\Gammaar{a}}$	$rac{ ilde{L}}{\Gamma \overline{a}}$	$rac{ ilde{L}}{2\Gammaar{a}}$	$\frac{\tilde{L}}{2\Gamma\bar{a}^2}$	$\frac{\tilde{L}}{\Gamma \bar{a}^2} + L_{Lb}$	$\frac{\tilde{L}}{2\Gamma\bar{a}^2}$
c	$rac{ ilde{L}}{\Lambda ar{a}}$	$rac{ ilde{L}}{2\Gammaar{a}}$	$\frac{\tilde{L}}{\bar{a}}$	$\frac{\tilde{L}}{\Lambda \bar{a}^2}$	$\frac{\tilde{L}}{2\Gamma\bar{a}^2}$	$\frac{\tilde{L}}{\bar{a}^2} + L_{Lc}$

Table 4.1: Transformer inductance matrix

- Core cross sectional area $[A_{CS}]$
- Number of winding turns of the high and low voltage windings $[N_X, N_x]$
- Mean yoke and limb lengths $[l_Y, l_E]$
- Leakage inductance of the high and low voltage windings $[L_{LX}, L_{Lx}]$

4.8 Conclusion

This chapter has derived the self and mutual inductances of a three phase two winding core type transformer. The inductance relationships were used to develop an inductance matrix for the transformer. Each inductive element is in terms of physical parameters for which later chapters will formulate estimates based on readily available transformer details. The inductance matrix is a fundamental component in the generic phase model developed in Chapter 6.

Chapter 5

Leakage Inductance

5.1 Introduction

With increasing frequency, the permeability of the core decreases [107], hence, the influence of the leakage inductance becomes more significant. It is therefore necessary to include leakage inductance in any wide-band frequency transformer model. Previous research regarding the estimation of leakage inductance for modelling, such as in [55], has not taken full advantage of the available test data and symmetry of a power transformer. Other authors [75], make the assumption that the high voltage winding leakage inductance accounts for somewhere between 75% and 90% of the total leakage inductance. In this chapter we formulate relationships which can be used to estimate the leakage inductance of a winding for use in a three phase transformer model of any vector group.

This chapter is structured in the following manner. Section 5.2 provides a brief overview on leakage inductance in power transformers. Section 5.3 discusses the determination of a transformer's percent reactance through short circuit testing. Percent reactance, together with other nameplate details, is used to determine the impedance of the power transformer. Section 5.4 investigates the leakage flux magnetic circuit and the relationship between the leakage flux of the primary and secondary windings. Section 5.5 uses transformer nameplate details to derive an estimate for the leakage inductance of the primary and secondary windings of a single phase transformer. Section 5.6 estimates the leakage inductance of the HV and LV windings of Dd, Dy, Yd and Yy connected three phase transformers. Concluding remarks are then given in Section 5.7.

5.2 Background

An important parameter for power system designers is transformer impedance. This impedance has both a resistive and a reactive component. The resistive component is due to resistance and eddy current based losses within the windings. This component also accounts for stray losses in structural parts of the transformer [71]. The reactance is due to the load current generating large magnetic fields around the respective windings.



Figure 5.1: Single phase equivalent circuit of a short circuit test used to determine a transformer's percent impedance.

These fields are commonly referred to as leakage flux fields and occupy the space both within and between the windings [52]. These fields can be thought of as an inductive impedance between the transformer windings, and are commonly referred to as leakage reactance. Though initially thought to be a shortcoming of transformer design, its value as a method of limiting short circuit currents has since been realised [53].

5.3 Short Circuit Test

A transformer's impedance is typically presented as a percentage that represents the voltage drop at full load current, relative to the rated voltage. Typically a transformer would have an impedance of approximately 2% for small distribution transformers and as high as 20% for large power transformers [71]. In practice this is determined by short circuiting either the primary or secondary winding, and increasing the voltage on the other winding until full load current is achieved. Figure 5.1 illustrates the test using a single phase equivalent circuit model.

During a short circuit test the magnetising current is small relative to the short circuit current, hence, it is assumed negligible [71]. The applied voltage v_{SC} is increased until full load current is achieved. At this point the percent impedance can be determined by expressing the applied voltage divided by the rated voltage as a percentage [52],

$$Z\% = \left|\frac{v_{SC}}{v_{HR}}\right| \times 100 \quad , \tag{5.1}$$

where v_{SC} is the voltage applied to the high voltage winding, v_{HR} is the rated voltage of the high voltage winding, and Z % is the percent impedance.

Further simplification of Figure 5.1 is made possible by referring the secondary impedances to the primary¹ side, (2.29) (2.30). The simplified short circuit test equivalent circuit is shown in Figure 5.2.

¹The primary is assumed to be the high voltage winding.


Figure 5.2: Simplified single phase equivalent circuit of a short circuit test.

With reference to Figure 5.2 it can be observed that,

$$v_{SC} = i_{SCX} Z_L = i_{HR} Z_L \quad , \tag{5.2}$$

where i_{SCX} is the HV short circuit current, i_{HR} is the rated current of the HV winding, and Z_L is the transformer impedance. During a short circuit test the current is increased until it is equal to the rated HV current i_{HR} . By combining (5.1) with (5.2),

$$Z\% = \left|\frac{i_{HR}Z_L}{v_{HR}}\right| \times 100 \quad . \tag{5.3}$$

A relationship for the impedance of a *single phase* power transformer in terms of its nameplate parameters is found by rearranging (5.3),

$$|Z_L| = \frac{v_{HR} Z \%}{100 i_{HR}} \quad . \tag{5.4}$$

From Figure 5.2 Z_L is given by,

$$Z_L = \left(R_X + \bar{a}^2 R_x\right) + jw \left(L_{LX} + \bar{a}^2 L_{Lx}\right) .$$
(5.5)

In the transformer equivalent circuits shown in Figures 5.1 and 5.2, the resistances, R_X and R_x , represent the resistive losses of the HV and LV windings respectively. However, relative to Z %, the contribution of the resistance is very small [71]. On this basis, it can be assumed that the leakage reactance parameters, L_{LX} and L_{Lx} , are the dominant contributors to the percent impedance and hence (5.5) can be approximated by,

$$Z_L \approx jw \left(L_{LX} + \bar{a}^2 L_{Lx} \right) \ . \tag{5.6}$$

Since the leakage reactance is much larger than the transformer resistance, the percent impedance is commonly referred to as percent leakage reactance [52, 53, 71].

With reference to the impedance (5.6), a commonly used approximation [51, 71] is to evenly distribute the impedance into a primary leakage reactance term and a "primary referred" secondary leakage reactance term, that is,

$$\frac{1}{2}Z_L \approx jwL_{LX} \approx jw\bar{a}^2 L_{Lx} \quad . \tag{5.7}$$

The following section investigates the merits of this approximation.

5.4 Leakage Flux Path

The magnetomotive force of each transformer winding is, generally, uniformly distributed along its axial length. Assuming that the windings are of similar height, the resulting leakage fields tend to be axial from one side of the core window to the other (neglecting fringing at the winding ends) [18, 71]. These axial flux lines meet the transformer core at either end of the core window resulting in a composite leakage reluctance that consists of the axial path across the core window, and the return path through the core,

$$\mathcal{R}_{LX} = \mathcal{R}_{WX} + \mathcal{R}_{FEX} , \qquad (5.8)$$

$$\mathcal{R}_{Lx} = \mathcal{R}_{Wx} + \mathcal{R}_{FEx} , \qquad (5.9)$$

where for the HV winding, \mathcal{R}_{LX} is the composite leakage reluctance, \mathcal{R}_{WX} is the core window leakage reluctance and \mathcal{R}_{FEX} is the magnetic core leakage reluctance. Likewise for the LV winding leakage reluctance.

The reluctance for a uniform magnetic circuit is given by,

$$\mathcal{R} = \frac{l}{\mu_0 \mu_r A} \quad , \tag{5.10}$$

where l is the flux path length, A is the cross sectional area of the flux path and μ_0 is the permeability of free space. The leakage flux traveling across the core window is in a region that is not ferromagnetic. As a result, the relative permeability tends to that of a vacuum, i.e. $\mu_r = 1$ [94], hence \mathcal{R}_{WX} and \mathcal{R}_{Wx} are frequency independent. In contrast, the permeability of a transformer core decays with increasing frequency due to magnetic skin effect (Chapter 3), resulting in an increase in the core reluctance. However, Chapter 3 also showed that for frequencies up to 1MHz, the permeability remains significant relative to a permeability of one. Therefore, for frequencies up to 1MHz an appropriate approximation is to assume that,

$$\mathcal{R}_{WX} \gg \mathcal{R}_{FEX}$$
, (5.11)

$$\mathcal{R}_{Wx} \gg \mathcal{R}_{FEx}$$
, (5.12)

and hence (5.8) and (5.9) can be approximated as,

$$\mathcal{R}_{LX} \approx \mathcal{R}_{WX}$$
, (5.13)

$$\mathcal{R}_{Lx} \approx \mathcal{R}_{Wx}$$
, (5.14)

where the leakage reluctance terms of (5.13) and (5.14) can be considered frequency independent.

Considering that the magnetising current is very small relative to the load current under short circuit test conditions, the HV magnetomotive force is approximately equal to that of the LV (2.19) [62],

$$N_X i_X \approx N_x i_x \quad , \tag{5.15}$$

where N_X is the number of HV winding turns and i_X is the HV winding current, and likewise for the LV winding. Noting that a transformer's low voltage winding generally occupies the position closest to the core, the leakage flux density will start from zero at the inside diameter of the low voltage winding and linearly increase to reach a maximum at its outside diameter. This maximum flux density will remain constant until the inside diameter of the high voltage winding, at which point the leakage flux density will linearly decay to zero at the outside diameter of the high voltage winding [52, 53, 71]. The resulting flux density profile is trapezoidal in shape when looking at a radial cross section of the windings [71, 94]. This is shown in Figure 5.3. This figure presents a transformer core window cross section where the LV winding is a helix wound high current winding and the HV winding is disc wound. It is noted that the flux lines are axially parallel with the peak leakage density between the windings and the flux density is zero beyond the windings. This figure is an idealisation only as all fringing effects have been neglected and it is assumed that the core is the return path for all flux lines. This would not be the case in practice as this leakage flux model simplifies the problem, however, it will provide a reasonable approximation.

For estimation purposes it is proposed that the leakage reluctance observed by the mmf of each winding be considered approximately equivalent, i.e

$$\mathcal{R}_{LX} \approx \mathcal{R}_{Lx}$$
 . (5.16)

Technically the reluctance values will not be equivalent since the leakage reluctance associated with each winding is dependent upon the geometry of the windings and their relationship with each other and the core. However with reference to (5.10) and Figure 5.3, two key observations can be made to support the approximation. The first is that the *axial* leakage path length across the core window is similar for both the HV and LV windings based on the assumption that the windings are of similar height. The second observation is that the leakage flux is concentrated in the area between the HV and LV windings.



Figure 5.3: Core window cross section detailing leakage flux density with respect to transformer high and low voltage windings (neglecting fringing at the winding ends).

The mmf associated with each winding will produce a flux proportional to the leakage reluctance path. From (5.15) and (5.16), the magnetic leakage flux of the HV winding is approximately equal to the magnetic leakage flux of the LV winding,

$$\phi_X \approx \phi_x \quad . \tag{5.17}$$

It is now possible to derive the relationship for leakage inductance of the two windings. From the self inductance definition (2.14),

$$L_{LX} = \frac{N_X \phi_X}{i_X} , \ L_{Lx} = \frac{N_x \phi_x}{i_x} .$$
 (5.18)

Utilising the assumption of balanced leakage flux (5.17), we have,

$$\frac{L_{LX}i_X}{N_X} \approx \frac{L_{Lx}i_x}{N_x}$$

$$L_{LX} \approx \frac{L_{Lx}N_Xi_x}{N_xi_X}$$

$$\therefore L_{LX} \approx \bar{a}L_{Lx}\left(\frac{i_x}{i_X}\right) .$$
(5.19)

For a single phase transformer (5.19) can be rewritten to take into account the current ratio (2.9),

$$L_{LX} \approx \overline{a}^2 L_{Lx} \quad , \tag{5.20}$$

which agrees with the relationship of (5.7).

It is proposed that this commonly used approximation [51, 71] is appropriate for its intended purpose².

As the leakage reactance is dependent upon the structural geometry of the transformer windings in relation to each other and the core, the greater the winding axial length, the greater the leakage reluctance, and hence the lower the leakage reactance [53], i.e,

$$Z\% \propto \frac{1}{l_{AW}} \tag{5.21}$$

where l_{AW} is the axial length of the winding. Therefore a tall thin transformer will have less flux leakage than an equivalent short wide transformer design.

5.5 Leakage Inductance of a Single Phase Transformer

This section will derive relationships which use nameplate details to estimate the HV and LV leakage inductance of a single phase transformer. Combining (5.7) with (5.4) for the

 $^{^{2}}$ The initial value estimate of HV and LV leakage inductance terms used in the model estimation algorithm of Chapter 7.

HV winding,

$$wL_{LX} \approx \frac{1}{2} \frac{v_{HR}Z\%}{100i_{HR}}$$

$$\therefore L_{LX} \approx \frac{v_{HR}Z\%}{400\pi f i_{HR}}, \qquad (5.22)$$

where f is mains frequency. Similarly for the LV winding from (5.7) and (5.4),

$$w\bar{a}^{2}L_{Lx} \approx \frac{1}{2} \frac{v_{HR}Z\%}{100i_{HR}}$$

$$\therefore L_{Lx} \approx \frac{v_{HR}Z\%}{400\pi f \bar{a}^{2}i_{HR}}.$$
(5.23)

This can be simplified further by noting the \bar{a} relationship for the single phase transformer current (2.27), which during a short circuit test is at rated value. Therefore (5.23) becomes,

$$L_{Lx} \approx \frac{v_{HR}Z\%}{400\pi f i_{HR}} \cdot \left(\frac{i_{HR}}{i_{LR}}\right)^2$$
$$\approx \frac{v_{HR}i_{HR}Z\%}{400\pi f i_{LR}^2} . \tag{5.24}$$

Relationships have now been derived to determine an initial estimate for the leakage inductance of the HV and LV windings of a single phase transformer, (5.22) and (5.24) respectively, utilising information available from a transformer nameplate. However, these relationships will not suffice for three phase systems since the vector group needs to be taken into account. This is investigated in the following section.

5.6 Leakage Inductance of a Three Phase Transformer

To obtain an approximation of the leakage inductance for each winding in a three phase transformer, it is necessary to take into account the vector group. In Section 2.9.3, a detailed analysis determined the per phase equivalent circuit for each vector group under consideration. With reference to Figure 2.15 and Table 2.4, a simplified per phase equivalent circuit for the short circuit test of each vector group, is presented in Figure 5.4. As discussed in Section 5.3, the resistance is very small, hence is neglected. There are two representations of the equivalent circuit, Figure 5.4 (a) is representative of the short circuit test of a Dd or Dy vector group and (b), of a Yd or Yy vector group.

Since the line to line open circuit voltages of a transformer are provided by the nameplate, with reference to the per phase equivalent circuit of Figure 5.4, the transformer



(a) Dd and Dy



(b) Yd and Yy

Figure 5.4: Simplified per phase equivalent circuit for a short circuit test with the secondary leakage inductance referred to the primary. (a) Dd and Dy vector groups. (b) Yd and Yy vector groups.

impedance definition of (5.1) can be rewritten. For a three phase application,

$$Z\% = \frac{\sqrt{3}|v_{SC}|}{v_{HR}} \times 100 , \qquad (5.25)$$

where v_{HR} is the high voltage nameplate rating, which is line to line, and v_{SC} is the phase to neutral voltage of the equivalent circuit (Figure 5.4). With reference to the equivalent circuit and noting that the short circuit test is conducted at rated current,

$$Z\% = \frac{\sqrt{3}i_{HR}|Z_L|}{v_{HR}} \times 100$$

: $|Z_L| = \frac{v_{HR}Z\%}{100\sqrt{3}i_{HR}}$, (5.26)

where i_{HR} is the nameplate line current rating for the high voltage windings, and Z_L represents the impedance. Hence a relationship has been obtained to calculate the magnitude of the impedance in terms of nameplate details.

The following sections will determine the leakage inductance relationships for each of the vector groups that are considered in this thesis.

5.6.1 Leakage Inductance Estimate for a Dd vector group transformer

With reference to Table 2.4, for a Dd vector group the transformer impedance is,

$$Z_L \approx jw \left(\frac{L_{LX}}{3} + \bar{a}^2 \frac{L_{Lx}}{3}\right) \quad . \tag{5.27}$$

As discussed in Section 5.3, a commonly used approximation [51, 71] is the equal division of the transformer impedance into a primary leakage reactance and a "primary referred" secondary leakage reactance. From (5.27),

$$\frac{1}{2}Z_L \approx jw\left(\frac{L_{LX}}{3}\right) \approx jw\bar{a}^2\left(\frac{L_{Lx}}{3}\right) \quad . \tag{5.28}$$

For the HV winding leakage inductance, substitute (5.26) into (5.28),

$$w\left(\frac{L_{LX}}{3}\right) \approx \frac{1}{2} \left[\frac{v_{HR}Z\%}{100\sqrt{3}i_{HR}}\right]$$

$$\therefore L_{LX} \approx \frac{3v_{HR}Z\%}{400\sqrt{3}\pi i_{HR}f}.$$
 (5.29)

For the LV winding leakage inductance, from Table 2.3, the transformer turns ratio for a Dd vector group can be determined from the current gain relationship K_i ,

$$|K_i| = \frac{i_{LR}}{i_{HR}} = \bar{a} \quad . \tag{5.30}$$

In this expression, i_{HR} and i_{LR} are the nameplate current ratings for the HV and LV windings respectively. Substituting (5.26) and (5.30) into (5.28),

$$w\left[\left(\frac{i_{LR}}{i_{HR}}\right)^2 \frac{L_{Lx}}{3}\right] \approx \frac{1}{2} \left[\frac{v_{HR}Z\%}{100\sqrt{3}i_{HR}}\right]$$

$$\therefore \ L_{Lx} \approx \frac{3v_{HR}i_{HR}Z\%}{400\sqrt{3}\pi i_{LR}^2 f} , \qquad (5.31)$$

Therefore, an initial estimate for the leakage inductance of the HV and LV windings, (5.29) and (5.31) respectively, can be determined for a Dd vector group transformer utilising readily available nameplate details.

5.6.2 Leakage Inductance Estimate for a Dy vector group transformer

With reference to Table 2.4, for a Dy vector group the transformer impedance is,

$$Z_L \approx jw \left(\frac{L_{LX}}{3} + \bar{a}^2 \frac{L_{Lx}}{3}\right) \quad . \tag{5.32}$$

Equally distributing the impedance of (5.32) into a primary leakage reactance and a "primary referred" secondary leakage reactance,

$$\frac{1}{2}Z_L \approx jw\left(\frac{L_{LX}}{3}\right) \approx jw\bar{a}^2\left(\frac{L_{Lx}}{3}\right) \quad . \tag{5.33}$$

For the HV winding leakage inductance, substitute (5.26) into (5.33),

$$w\left(\frac{L_{LX}}{3}\right) \approx \frac{1}{2} \left[\frac{v_{HR}Z\%}{100\sqrt{3}i_{HR}}\right]$$

$$\therefore L_{LX} \approx \frac{3v_{HR}Z\%}{400\sqrt{3}\pi i_{HR}f}.$$
 (5.34)

For the LV winding leakage inductance, from Table 2.3, the transformer turns ratio for a Dy vector group can be determined from the current gain relationship K_i ,

$$|K_i| = \frac{i_{LR}}{i_{HR}} = \frac{\bar{a}}{\sqrt{3}}$$

$$\therefore \ \bar{a} = \frac{\sqrt{3}i_{LR}}{i_{HR}} .$$
(5.35)

Substituting (5.26) and (5.35) into (5.33),

$$w\left[\left(\frac{\sqrt{3}i_{LR}}{i_{HR}}\right)^2 \frac{L_{Lx}}{3}\right] \approx \frac{1}{2} \left[\frac{v_{HR}Z\%}{100\sqrt{3}i_{HR}}\right]$$

$$\therefore L_{Lx} \approx \frac{v_{HR}i_{HR}Z\%}{400\sqrt{3}\pi i_{LR}^2 f}.$$
 (5.36)

Here (5.34) and (5.36) provide an initial estimate for the leakage inductance of the HV and LV windings respectively, of a Dy vector group transformer.

5.6.3 Leakage Inductance Estimate for a Yd vector group transformer

With reference to Table 2.4, for a Yd vector group the transformer impedance is,

$$Z_L \approx jw \left(L_{LX} + \bar{a}^2 L_{Lx} \right) \quad . \tag{5.37}$$

Equally distributing the impedance of (5.37) into a primary leakage reactance and a "primary referred" secondary leakage reactance,

$$\frac{1}{2}Z_L \approx jwL_{LX} \approx jw\bar{a}^2 L_{Lx} \quad . \tag{5.38}$$

For the HV winding leakage inductance, substitute (5.26) into (5.38),

$$wL_{LX} \approx \frac{1}{2} \left[\frac{v_{HR}Z\%}{100\sqrt{3}i_{HR}} \right]$$

$$\therefore L_{LX} \approx \frac{v_{HR}Z\%}{400\sqrt{3}\pi i_{HR}f} . \qquad (5.39)$$

For the LV winding leakage inductance, from Table 2.3, the transformer turns ratio for a Yd vector group can be determined from the current gain relationship K_i ,

$$|K_i| = \frac{i_{LR}}{i_{HR}} = \bar{a}\sqrt{3}$$

$$\therefore \ \bar{a} = \frac{i_{LR}}{\sqrt{3}i_{HR}} .$$
(5.40)

Substituting (5.26) and (5.40) into (5.38),

$$w \left[\left(\frac{i_{LR}}{\sqrt{3} i_{HR}} \right)^2 L_{Lx} \right] \approx \frac{1}{2} \left[\frac{v_{HR} Z \%}{100 \sqrt{3} i_{HR}} \right]$$

$$\therefore L_{Lx} \approx \frac{3 v_{HR} i_{HR} Z \%}{400 \sqrt{3} \pi i_{LR}^2 f} , \qquad (5.41)$$

Equations (5.39) and (5.41) provide an initial estimate for the leakage inductance of the HV and LV windings respectively, of a Yd vector group transformer.

5.6.4 Leakage Inductance Estimate for a Yy vector group transformer

With reference to Table 2.4, for a Yy vector group the transformer impedance is,

$$Z_L \approx jw \left(L_{LX} + \bar{a}^2 L_{Lx} \right) \quad . \tag{5.42}$$

Equally distributing the impedance of (5.42) into a primary leakage reactance and a "primary referred" secondary leakage reactance,

$$\frac{1}{2}Z_L \approx jwL_{LX} \approx jw\bar{a}^2 L_{Lx} \quad . \tag{5.43}$$

For the HV winding leakage inductance, substitute (5.26) into (5.43),

$$wL_{LX} \approx \frac{1}{2} \left[\frac{v_{HR} Z \%}{100 \sqrt{3} i_{HR}} \right]$$

$$\therefore L_{LX} \approx \frac{v_{HR} Z \%}{400 \sqrt{3} \pi i_{HR} f} . \qquad (5.44)$$

For the LV winding leakage inductance, from Table 2.3, the transformer turns ratio

Vector Group	L_{LX}	L_{Lx}
Dd	$3\hat{L}_{LX}$	$3\hat{L}_{Lx}$
Dy	$3\hat{L}_{LX}$	\hat{L}_{Lx}
Yd	\hat{L}_{LX}	$3\hat{L}_{Lx}$
Yy	\hat{L}_{LX}	\hat{L}_{Lx}

Table 5.1: Three phase leakage inductance initial estimates

for a Yy vector group can be determined from the current gain relationship K_i ,

$$|K_i| = \bar{a} = \frac{i_{LR}}{i_{HR}} \tag{5.45}$$

Substituting (5.26) and (5.45) into (5.43),

$$w\left[\left(\frac{i_{LR}}{i_{HR}}\right)^2 L_{Lx}\right] \approx \frac{1}{2} \left[\frac{v_{HR}Z\%}{100\sqrt{3}i_{HR}}\right]$$

$$\therefore L_{Lx} \approx \frac{v_{HR}i_{HR}Z\%}{400\sqrt{3}\pi i_{LR}^2 f}.$$
 (5.46)

Equation (5.44) and (5.46) provide an initial estimate for the leakage inductance of the HV and LV windings respectively, of a Yy vector group transformer.

5.6.5 Winding Leakage Inductance Estimate Summary

A summary of the leakage inductance estimate equations is given in Table 5.1 where

$$\hat{L}_{LX} = \frac{v_{HR}Z\%}{400\sqrt{3}\pi i_{HR}f}$$
 and (5.47)

$$\hat{L}_{Lx} = \hat{L}_{LX} \left(\frac{i_{HR}}{i_{LR}}\right)^2 .$$
(5.48)

In equations (5.47) and (5.48), \hat{L}_{LX} is the nominal high voltage winding leakage inductance and \hat{L}_{Lx} is the nominal low voltage winding leakage inductance for generic phase X. v_{HR} is the transformer high voltage rating, i_{HR} and i_{LR} are the high and low voltage terminal current ratings, Z % is the percent reactance and f is the operating frequency. It is clear from Table 5.1 that, like the delta/star transformation relationship of (2.66), the estimate, based on the associated terminal ratings, involves a factor of three for a delta connection.

5.7 Conclusion

In power system analysis it is convenient to reduce the complexity of a network by considering three phase transformers on a per phase basis. The transformer details required for this analysis are obtained from the transformer nameplate. This information is also valuable from the perspective of transformer modelling. In this chapter, the percent reactance and other details from the nameplate were used to formulate leakage inductance relationships for the high and low voltage windings of several vector group topologies. The relationships developed in this chapter are used to determine initial values for the leakage inductance parameters used in the parameter estimation algorithm implemented in Chapter 9. By using genuine approximations of parameter values in an estimation algorithm, problems such as local minima can be avoided, helping to ensure that the final parameter estimates are physically feasible.

Chapter 6

Generic Phase Model

6.1 Introduction

This chapter develops a generic phase, wide-band frequency, transformer model. The model incorporates the complex permeability of the core, as developed in Chapter 3, and the self and mutual inductance relationships developed in Chapter 4. It also takes into account the major capacitive coupling influences as well as the loss terms associated with the skin and proximity effect of the winding. Where possible, each model component is reduced into a physically quantifiable parameter, e.g. core dimensions, number of winding turns and lamination thickness. This approach will enable the veracity of the model to be demonstrated by comparing estimated parameters to those that can be physically measured.

This chapter is structured in the following manner. Section 6.2 details the use of generic phase and section references within the modelling approach. Section 6.3 develops a frequency dependent generic inductance model based on the work from Chapters 3 and 4. Section 6.4 develops a frequency dependent winding resistance model that incorporates the effects of skin and proximity effect. Section 6.5 derives analytical estimates based on geometry for each of the model's capacitor elements. Section 6.6 constructs the generic phase model from the circuit elements developed in the previous sections. Concluding remarks are given in Section 6.7.

6.2 Generic Phase Referencing

An FRA test measures the frequency response between a nominal pair of transformer terminals. There are a number of different FRA test types [6], however, for a three phase transformer, each test sequence will typically record three measurements. For example, in a High Voltage End to End FRA test, measurements are taken between A-B, B-C and C-A¹. To construct a model that can be representative of different terminal pair combinations, it is convenient to utilise generic phase references. As discussed in Section

¹FRA test connections are discussed in detail in Chapter 7.



Figure 6.1: Composite inductor element i for generic HV winding X

2.3.3, this research proposes the use of X-Y-Z for the HV terminals and x-y-z for the LV terminals.

The use of generic phase referencing facilitates the substitution of the physical attributes associated with each phase for different test combinations. For example, if generic phase X is assigned to phase B for one test, then all of the geometric details associated with phase B are used to calculate the parameters within the generic phase X model during the modelling of that particular test. Likewise for assignments to generic phase models Y and Z.

In addition, since a lumped parameter model is made up of many "sections", subscripts i and j are used to represent generic winding sections within the n section generic model.

For the remainder of this chapter discussion will tend to focus on generic phase X. However, all modelling in this chapter applies equally to generic phases Y and Z.

6.3 Inductance Model

In Chapter 4, self and mutual inductance relationships were derived for a three phase core type transformer (see Table 4.1 for a summary). The relationships comprised of both ferromagnetic and leakage inductance components. The ferromagnetic component of each entry in Table 4.1 is formulated around the base inductance, \tilde{L} . The leakage inductance for the high voltage winding is given by L_{LX} , and the low voltage winding, L_{Lx} . The composite inductance element², \mathfrak{L}_{Xi} , is shown in Figure 6.1. The contributions of the ferromagnetic and leakage components are discussed separately in the following sections.

²The composite nature of the inductive element is symbolised by the use of the Fraktur character \mathcal{L} .

L	A_j	B_j	C_j	a_j	b_j	c_j
A_i	$\bar{L} + L_{LAij}$	$rac{ar{L}}{2\Gamma}$	$\frac{\bar{L}}{\Lambda}$	$\frac{\bar{L}}{\bar{a}}$	$rac{ar{L}}{2\Gammaar{a}}$	$rac{ar{L}}{\Lambdaar{a}}$
B_i	$rac{ar{L}}{2\Gamma}$	$\frac{\bar{L}}{\Gamma} + L_{LBij}$	$rac{ar{L}}{2\Gamma}$	$rac{ar{L}}{2\Gammaar{a}}$	$rac{ar{L}}{\Gammaar{a}}$	$rac{ar{L}}{2\Gammaar{a}}$
C_i	$\frac{\bar{L}}{\Lambda}$	$rac{ar{L}}{2\Gamma}$	$\bar{L} + L_{LCij}$	$\frac{\bar{L}}{\Lambda \bar{a}}$	$rac{ar{L}}{2\Gammaar{a}}$	$\frac{\bar{L}}{\bar{a}}$
a_i	$rac{ar{L}}{ar{a}}$	$rac{ar{L}}{2\Gammaar{a}}$	$rac{ar{L}}{\Lambdaar{a}}$	$\frac{\bar{L}}{\bar{a}^2} + L_{Laij}$	$rac{ar{L}}{2\Gammaar{a}^2}$	$rac{ar{L}}{\Lambdaar{a}^2}$
b_i	$rac{ar{L}}{2\Gammaar{a}}$	$rac{ar{L}}{\Gammaar{a}}$	$rac{ar{L}}{2\Gammaar{a}}$	$rac{ar{L}}{2\Gammaar{a}^2}$	$\frac{\bar{L}}{\Gamma \bar{a}^2} + L_{Lbij}$	$rac{ar{L}}{2\Gammaar{a}^2}$
c_i	$rac{ar{L}}{\Lambdaar{a}}$	$rac{ar{L}}{2\Gammaar{a}}$	$\frac{\bar{L}}{\bar{a}}$	$rac{ar{L}}{\Lambdaar{a}^2}$	$rac{ar{L}}{2\Gammaar{a}^2}$	$\frac{\bar{L}}{\bar{a}^2} + L_{Lcij}$

Table 6.1: Sectional transformer inductance matrix

6.3.1Ferromagnetic Inductance

The ferromagnetic base inductance, \tilde{L} , as derived in Chapter 4 (4.62), is representative of an entire winding. When considered from the perspective of an n section lumped parameter model, the number of turns per section is given by,

for the high voltage winding, for the low voltage winding.

From (4.62), the ferromagnetic base inductance for a model that is comprised of nsections is given by, 0

$$\bar{L} = \frac{\tilde{L}}{n^2} = \frac{\mu A_{CS} \left(\frac{N_X}{n}\right)^2 (2l_E + l_Y)}{(l_E + l_Y) (3l_E + l_Y)} , \qquad (6.1)$$

where μ is the complex permeability, A_{CS} is the core cross sectional area, l_E is the limb length and l_Y is the yoke length. Note that the complex permeability is defined as,

$$\mu = \mu_0 \mu_s \tag{6.2}$$

where μ_0 is the permeability of free space and μ_s is the effective permeability (3.22).

Considering the ferromagnetic contribution to the inductances as defined in Table 4.1, L is weighted by the transformer turns ratio \bar{a} , and the transformer core length ratios,

$$\Gamma = \frac{2l_E + l_Y}{2(l_E + l_Y)} , \qquad (6.3)$$

$$\Lambda = \frac{2l_E + l_Y}{l_E} . \tag{6.4}$$

An n section version of Table 4.1 is presented in Table 6.1. Each entry in the table is frequency dependent with μ as per (3.22). The leakage inductance is discussed in Section 6.3.2.

Due to the effective permeability being complex, (3.22), the resulting expression can be considered to be a combination of inductive and resistive components (3.24). The inductive component is a combination of self inductance, $L_{Xi}(\omega)$, and its affiliated mutual inductance. The frequency dependent resistance, $R_{Xi}(\omega)$, is considered to be the eddy current loss component of the core. Since the flux generated within the ferromagnetic core of the transformer is assumed to have no leakage, the ferromagnetic mutual inductance between sections of the same winding is considered equivalent to the self inductance of the winding, i.e.

$$L_{Xi}(\omega) = L_{XiXj}(\omega) \tag{6.5}$$

$$L_{xi}(\omega) = L_{xixj}(\omega) . ag{6.6}$$

The ferromagnetic contribution to the composite inductance element can be observed in Figure 6.1.

6.3.2 Leakage Inductance

The leakage inductance is due to flux fields both within and between the associated windings. Some of the leakage flux will link all turns of the winding that generates it, whereas some will only link a proportion of them [103], i.e. leakage coupling is strongest between turns in close proximity. On this basis, when considering leakage inductance in an nsection lumped parameter model, the mutual coupling between leakage inductance elements across the winding is non-uniform. It is therefore necessary to introduce a coupling coefficient that decreases with respect to the axial distance between circuit elements.

As discussed in Section 5.4, leakage flux travels through the transformer windings and returns via the core. As a result the path reluctance is dominated by the relative permeability of the materials within the winding window which approaches unity. As a consequence, we consider the leakage coupling coefficient versus axial distance profile for each winding to approximate that of an air cored solenoid. In the seminal papers by Wilcox et al. [122][123], expressions were developed for the calculation of the self and mutual impedances associated with transformer winding sections. Part of the work by Wilcox et al. [123] considered the inductance between winding section i and j in air, as shown in Figure 6.2. The equation for the air cored inductance between these two sections, \tilde{L}_{ij} , for an axial distance z, is given by,

$$\tilde{L}_{ij} = \mu_0 N_i N_j \sqrt{(r_i r_j)} \frac{2}{\kappa} \left[\left(1 - \frac{\kappa^2}{2} \right) K(\kappa) - E(\kappa) \right] , \qquad (6.7)$$

where N_i , N_j are the number of turns for the respective winding sections and r_i , r_j are the mean sectional radii. $K(\kappa)$ and $E(\kappa)$ are the complete elliptic integrals of the first and second kind where κ is given by,

$$\kappa = \sqrt{\frac{4r_i r_j}{z^2 + (r_i + r_j)^2}} \quad . \tag{6.8}$$

The equation defined in (6.7) is not directly applicable to the distributed leakage induc-



Figure 6.2: Winding sections i and j in air

tance that exists within the primary and secondary windings of a transformer. Here we propose that the decay profile of the mutual inductance between two winding sections versus the distance between them in air, as governed by (6.7), is an appropriate decay profile to model the inductive coupling of distributed leakage inductance sections versus axial distance. For example, using (6.7) we can obtain the mutual inductance versus distance profile for two 325mm radius, 100 turn winding sections, as shown in Figure 6.3.

It is proposed that the decay of the mutual (leakage) inductance with respect to distance can be modelled by a cascaded coupling coefficient where³,

$$L_{LXij} \approx \tau^{|i-j|} L \quad \text{for } 0 \le \tau \le 1 \text{ and } i, j \in [1..n].$$

$$(6.9)$$

In (6.9), L_{LXij} is the mutual (leakage) inductance between winding sections *i* and *j* of generic phase *X*, *L* is the self (leakage) inductance of an individual section, and τ is the model coupling coefficient where $\tau = 0$ is no coupling and $\tau = 1$ is perfect coupling. A 4 section model is shown in Figure 6.4.

To demonstrate the applicability of the cascaded coupling coefficient, the mutual inductance versus section distance, for a 20 section model, is fitted to the Wilcox equation profile of Figure 6.3. This is accomplished by determining a coupling coefficient τ such that $L_{LXij} \approx \tilde{L}_{ij}$ across equivalent axial and sectional distances. In this example the best fit was obtained with $\tau = 0.76$. The sectional distance is scaled by equating the number of sections n to the maximum axial distance z_{MAX} (n = 20, $z_{MAX} = 1000$ mm).

From the cascaded coupling coefficient (6.9), the leakage inductance of the phase X winding, L_{LX} , can be determined through the summation of all the individual self and

 $^{^{3}}$ The HV winding leakage inductance is used as the reference for the remainder of this section, however the relationships developed here equally apply to the LV winding.



Figure 6.3: (Solid line) Mutual inductance versus axial distance of two winding sections in air. (Dotted line) Mutual (leakage) inductance versus winding sectional distance with $\tau = 0.76$.



Figure 6.4: Cascaded coupling coefficient leakage inductor model (n = 4)

6.4. WINDING RESISTANCE MODEL

mutual leakage inductance terms,

$$L_{LX} \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \tau^{|i-j|} L \quad \text{for } 0 \le \tau \le 1 \text{ and } i, j \in [1..n].$$
(6.10)

Given (6.9) and (6.10), a relationship for determining the leakage inductance between sections i and j in terms of the overall leakage inductance, L_{LX} , can now be derived,

$$L_{LXij} \approx \frac{\tau^{|i-j|} L_{LX}}{\left[\sum_{i=1}^{n} \sum_{j=1}^{n} \tau^{|i-j|}\right]} \text{ for } 0 \le \tau \le 1 \text{ and } i, j \in [1..n].$$
(6.11)

Using an estimate for the overall leakage inductance L_{LX} , as detailed in Section 5.6, and the knowledge that τ is bounded between 0 and 1, constraints can be placed on L_{LXij} when it is used in the parameter estimation algorithm of Chapter 9.

The leakage inductance contribution to the composite inductance circuit element is shown in Figure 6.1 where L_{LXi} is the leakage self inductance of section *i*, i.e. i = j, and L_{LXij} is the mutual leakage inductance between sections *i* and *j*.

6.4 Winding Resistance Model

The transformer winding can be considered to have an inherent DC and a frequency dependent AC resistance. The DC resistance is directly proportional to the resistivity of the conductor which is inversely proportional to the winding conductor cross sectional area. In a typical power transformer, the large conductor cross sectional area ensures that the DC resistance, R_{DC} , is comparatively very small.

AC resistance is due to the induction of eddy currents within the windings. These induced resistive losses can be classified into two categories, skin and proximity effect.

6.4.1 AC Resistance - Skin Effect

Skin effect is due to the magnetic field generated by the current in the conductor. This has the effect of increasing the current density near the conductor surface relative to its centre, which results in an increase in the effective resistance.

An estimate for the resistance due to skin effect can be made using the Dowell Method [42]. Assuming that the conductors are closely packed, each layer of a winding will approximate the geometry of a conductor foil, hence, the problem can be reduced to a one dimensional model.

The AC resistance due to skin effect [48], is given by,

$$R_S = \frac{R_{DC}\xi}{2} \left[\frac{\sinh \xi + \sin \xi}{\cosh \xi - \cos \xi} \right], \qquad (6.12)$$

where

$$\xi = \frac{d\sqrt{\pi}}{2\delta} , \qquad (6.13)$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \tag{6.14}$$

is the skin depth, d is the conductor diameter, f the frequency in Hz, and the permeability and conductivity of the conductor material are given by μ and σ respectively. Given a conductor cross sectional area A_X , the conductor diameter in (6.13) can be estimated to be,

$$d \approx 2\sqrt{\frac{A_X}{\pi}} \quad . \tag{6.15}$$

Substituting (6.15) into (6.13) we get

$$\xi = \frac{\sqrt{A_X}}{\delta} \ . \tag{6.16}$$

6.4.2 AC Resistance - Proximity Effect

Proximity effect occurs when the current in adjacent conductors generate magnetic fields which induce circulating eddy currents in neighbouring conductors. This will impact on the conductors current distribution in the same manner as skin effect, and therefore will increase the effective resistance.

Again, assuming that the conductors are closely packed and that each layer of a winding approximates the geometry of a conductor foil, the AC resistance due to proximity effect for the mth layer [48] is,

$$R_P = \frac{R_{DC}\xi}{2} \left[(2m-1)^2 \frac{\sinh \xi - \sin \xi}{\cosh \xi + \cos \xi} \right].$$
 (6.17)

6.4.3 Composite Winding Resistance

Assuming that the magnetic field from the other conductors is uniform across the conductor cross section, an orthogonal relationship exists between the skin and proximity effect [85]. The two effects can be decoupled and an estimate for the total eddy current losses can be determined through the addition of both effects (6.12) and (6.17). Combining both the AC and DC winding loss contributions and considering them to be sectionally distributed, the composite resistance element⁴, \Re_{Xi} , is given by,

$$\Re_{Xi}(\omega) = R_{PXi}(\omega) + R_{SXi}(\omega) + R_{DCXi} , \qquad (6.18)$$

and is shown in Figure 6.5.

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⁴The composite nature of the resistance element is symbolised by the use of the Fraktur character \Re .



Figure 6.5: Composite series resistance element i for generic winding X

The DC resistance R_{DC} is a measurable quantity. The permeability of copper approximates that of free space and the conductivity is typically $\sigma = 58 \times 10^6$ S/m. With this information, given parameter estimates for the number of winding layers m and the conductor cross sectional area A_X , a value for the composite series resistance element $\Re_{Xi}(\omega)$ can be estimated.

6.5 Capacitance Model

There are many forms of capacitive coupling within a transformer. A transformer cross sectional view given in Figure 6.6 shows several important capacitive elements in the transformer structure. To obtain a physically representative transformer model across a broad range of frequencies, it is important to incorporate all of the dominant capacitive elements into the model.

In the following sections we discuss each of the capacitances used in the model and derive relationships, based on transformer dimensions, in order to facilitate an estimate of their value. The derived relationships are used in conjunction with finite element analysis to validate the results in Chapter 9. This section on capacitance concludes with a discussion of the non-ideal nature of capacitors and the need for the capacitive element to include a parallel loss path [52].

6.5.1 High Voltage to Low Voltage Winding Capacitance

The capacitance between the high and low voltage windings of a transformer can be estimated by treating it as a cylindrical capacitor. To determine this capacitance we assume that a charge of +Q and -Q is present on each of the capacitor's conductor surfaces. These charges result in the formation of an electric field intensity E. Gauss's Law, given by,

$$\oint_{s} \mathbf{D}.ds = Q \quad , \tag{6.19}$$

where **D** is the electric flux density and Q is the total free charge enclosed on the surface. Now **D** = ϵ **E** where ϵ is the electrical permittivity of the dielectric medium between the two cylindrical conductors, hence (6.19) can be written as,



Figure 6.6: Cross sectional top view of a three phase two winding power transformer detailing interphase, interwinding, winding to tank and winding to core capacitance.



Figure 6.7: Cylindrical capacitor.

$$\oint_{s} \mathbf{E}.ds = \frac{Q}{\epsilon} \quad . \tag{6.20}$$

With reference to Figure 6.7, the inside radius of the outer conductor is r_o and the outside radius of the inner conductor is r_i . Using cylindrical coordinates with unitary vector **a**, the electric field intensity within the dielectric at radius r, where $r_i < r < r_o$, is given by [126, 32],

$$\mathbf{E} = \mathbf{a_r} \frac{Q}{\epsilon(2\pi rL)} \quad , \tag{6.21}$$

where L is the conductor length. We next determine the potential difference between the two conductors,

$$v = -\int_{r_i}^{r_o} \mathbf{E}.dl$$

= $-\int_{r_i}^{r_o} \left[\mathbf{a_r} \frac{Q}{(2\pi\epsilon rL)} \right] \cdot \left[\mathbf{a_r} dr \right]$
= $\frac{Q}{(2\pi\epsilon L)} \ln \left[\frac{r_o}{r_i} \right]$ (6.22)

The capacitance is then given by,

$$C = \frac{Q}{v} = \frac{2\pi\epsilon L}{\ln\left[\frac{r_o}{r_i}\right]} \quad . \tag{6.23}$$

With reference to Figure 6.6, (6.23) can be used as an estimate for the sectional



Figure 6.8: Parallel cylinders.

high voltage winding to low voltage winding capacitance, C_{Xxi} , in terms of transformer dimensions, i.e.

$$C_{Xxi} \approx \frac{2\pi\epsilon L}{n\ln\left[\frac{\Theta_{HID}}{\Theta_{LOD}}\right]} ,$$
 (6.24)

Where Θ_{HID} is the high voltage winding inside diameter and Θ_{LOD} the low voltage winding outside diameter. Note that (6.24) is based on *n* sections.

6.5.2 Low Voltage Winding to Core Capacitance

The capacitance of the low voltage winding to transformer core can also be treated as a cylindrical capacitor. With reference to Figure 6.6 and equation (6.23), an estimate for the sectional capacitance, C_{gxi} , in terms of transformer dimensions is,

$$C_{gxi} \approx \frac{2\pi\epsilon L}{n\ln\left[\frac{\Theta_{LID}}{\Theta_{COD}}\right]} ,$$
 (6.25)

where Θ_{LID} is the low voltage winding inside diameter and Θ_{COD} the transformer's core outside diameter. This capacitance is based on *n* sections.

6.5.3 Interphase High Voltage Winding Capacitance

The capacitance between the high voltage windings of two adjacent phases can be estimated by treating the geometry as two parallel cylinders as shown in Figure 6.8 [71]. The capacitance per unit length for this geometry [71, 126, 32], is given by,

$$C = \frac{\pi \epsilon}{\cosh^{-1} \left[\frac{S}{R}\right]} \quad , \tag{6.26}$$

6.5. CAPACITANCE MODEL

where 2S is the distance between the cylinder centres and R is the radius of both cylinders. Relating the dimensions of Figure 6.8 to those of Figure 6.6,

$$S = \frac{\Theta_{HOD} + \delta_{HV}}{2} \tag{6.27}$$

and

$$R = \frac{\Theta_{HOD}}{2} \quad , \tag{6.28}$$

where δ_{HV} is the smallest distance between two adjacent high voltage windings and Θ_{HOD} is the outside diameter of the high voltage winding. The capacitance (per unit length) in terms of the transformer dimensions can be determined by substituting (6.27) and (6.28) into (6.26), hence,

$$C = \frac{\pi\epsilon}{\cosh^{-1}\left[1 + \frac{\delta_{HV}}{\Theta_{HOD}}\right]} \quad (6.29)$$

Now defining C_{XYi} to be the generic capacitance of each section between the high voltage windings of two adjacent phases, for a winding of length L with n winding sections, we have,

$$C_{XYi} \approx \frac{\pi \epsilon L}{n \cosh^{-1} \left[1 + \frac{\delta_{HV}}{\Theta_{HOD}} \right]}$$
 (6.30)

This relationship in conjunction with an estimate of the transformer's HV winding diameter and core dimensions, as detailed in Chapter 8, can be used to place constraints on model estimates of C_{XYi} .

6.5.4 High Voltage Winding to Tank Capacitance

The capacitance between the high voltage winding and the tank can be estimated by considering the transformer as a cylindrical conductor with respect to a ground plane [71]. With reference to Figure 6.9, the relationship for a cylinder to ground plane capacitance per unit length [71, 126] is,

$$C = \frac{2\pi\epsilon}{\cosh^{-1}\left[\frac{S}{R}\right]} \quad , \tag{6.31}$$

where S is the distance between the centre of the cylinder and the plane, and R is the outside radius of the cylinder. With reference to Figure 6.6, (6.31) can be related to the high voltage winding to tank capacitance of each section, C_{gXi} , in terms of the transformer dimensions,



Figure 6.9: Cylindrical conductor with respect to a ground plane.

$$C_{gXi} \approx \frac{2\pi\epsilon L}{n\cosh^{-1}\left[1 + \frac{2\Delta_{SW}}{\Theta_{HOD}}\right]}$$
, (6.32)

where L is the high voltage winding axial length, Δ_{SW} is the orthogonal distance between the outside diameter of the high voltage winding and the transformer tank side walls, and Θ_{HOD} is the outside diameter of the high voltage winding.

It can be observed from Figure 6.6 that in practice the high voltage winding to tank capacitance of phase B would be subtly smaller than that of phases A and C due to the influence of the end walls of the tank. This influence was not considered in (6.32).

As with the interphase capacitance relationship, (6.32) in conjunction with the transformer external dimensions and an estimate of its HV winding diameter and core dimensions, as detailed in Chapter 8, can be used to place constraints on model estimates of C_{qXi} .

6.5.5 Series Winding Capacitance

The term series capacitance is a general term that includes the capacitance between turns in the same disc as well as the capacitance between turns in adjacent discs. This is shown in Figure 6.10 with C_{RT} denoting the radial turn to turn capacitance and C_{AT} the axial turn to turn capacitance. Series Capacitance is of critical importance in transformer design since its distributed value is a determining factor in the initial voltage distribution, and hence the resulting insulation stress, that occurs during a transient over-voltage [53]. When the series capacitance is large relative to the capacitance to ground, the initial voltage is more evenly distributed, therefore the response to a voltage surge is more benign



Figure 6.10: Series Capacitance within a disc pair of a continuous disc winding.



Figure 6.11: High voltage winding disc pair.

[52]. As a result, a number of approaches have been taken to increase the effective series capacitance by altering the arrangement of turns within a disc section of a transformer. Typical approaches taken include electrostatic shielding and the interleaving of turns [53]. Here we focus on the continuous and interleaved disc winding approaches.

Axial Turn Capacitance

The sectional approach to modelling a transformer divides the windings into many sections. For each of these sections, the axial dimensions can be considered small relative to a transformer's winding diameter. As a result, the use of simple parallel plate formulae can be used to derive the capacitance between two consecutive discs [52]. With reference to Figure 6.11 and assuming uniform permittivity, the axial capacitance is given by [38],

$$C_{AD} = \frac{\epsilon \pi \left(\Theta_{HOD}^2 - \Theta_{HID}^2\right)}{4 d_S} , \qquad (6.33)$$

where C_{AD} is the axial capacitance between two adjacent discs, d_S is the axial distance between the discs, and Θ_{HOD} and Θ_{HID} are the outside and inside diameters respectively.

With reference to Figure 6.10 it can be observed that a disc is comprised of $(N_{TD} - 1)$ axial turn capacitances, C_{AT} , where N_{TD} is the number of turns per disc. From (6.33),

$$C_{AT} = \frac{C_{AD}}{(N_{TD} - 1)}$$
$$= \frac{\epsilon \pi \left(\Theta_{HOD}^2 - \Theta_{HID}^2\right)}{4 d_S \left(N_{TD} - 1\right)} . \tag{6.34}$$

Radial Turn Capacitance

The radial turn capacitance C_{RT} , as shown in Figure 6.10, can also be determined using a parallel plate approach by treating two adjacent turns as parallel conductors [38]. This results in,

$$C_{RT} = \epsilon \left[\frac{\pi w D_m}{t_p} \right] \quad , \tag{6.35}$$

where w is the conductor width, D_m is the mean winding diameter and t_P is twice the paper insulation thickness. Incorporating the transformer dimensions of Figure 6.6 into (6.35),

$$C_{RT} = \frac{\epsilon \pi w \left[\Theta_{HOD} + \Theta_{HID}\right]}{2t_p} \quad . \tag{6.36}$$

Equivalent Series Capacitance of the Continuous Disc Winding

The equivalent series capacitance can store the same amount of electrostatic energy that a winding section would store if the section had a linearly distributed voltage applied across it [36]. Applying the voltage v_{DP} to the disc pair shown in Figure 6.10, and assuming that the voltage is distributed evenly across all of the $2N_{TD}$ turns of the disc pair, then the voltage per turn is,

$$v_{RT} = \frac{v_{DP}}{2N_{TD}} \quad . \tag{6.37}$$

By equating the electrostatic energy across all of the radial turn capacitances to an equivalent value for the disc pair C_{ER} [71],

$$\frac{1}{2}C_{ER}v_{DP}^2 = \frac{1}{2}C_{RT}\left[\frac{v_{DP}}{2N_{TD}}\right]^2 \left[2\left(N_{TD}-1\right)\right] \quad , \tag{6.38}$$

as there are $2(N_{TD}-1)$ radial turn capacitances per disc pair. Arranging (6.38) in terms of C_{ER} , we have

$$C_{ER} = \frac{(N_{TD} - 1)C_{RT}}{2N_{TD}^2} \quad . \tag{6.39}$$

Given that (6.39) is for a disc pair, the equivalent radial capacitance for a winding section C_{ERX} is,

$$C_{ERX} = \frac{(N_{TD} - 1)C_{RT}}{N_{DS}N_{TD}^2} , \qquad (6.40)$$

where N_{DS} is the number of discs per section. A relationship in terms of transformer dimensions, Figure 6.6, can be found by substituting (6.36) into (6.40),

$$C_{ERX} = \frac{\epsilon \pi w (N_{TD} - 1) (\Theta_{HOD} + \Theta_{HID})}{2 t_p N_{DS} N_{TD}^2}$$

$$\approx \frac{\epsilon \pi w (\Theta_{HOD} + \Theta_{HID})}{2 t_p N_{DS} N_{TD}} , \qquad (6.41)$$

noting that $N_{TD} \gg 1$.

The voltage across each of the axial turn capacitors, C_{AT} , can now be determined from the radial capacitor voltage distribution. Starting at the inside diameter, the sequence is [71],

$$\frac{2v_{DP}}{2N_{TD}}, \ \frac{4v_{DP}}{2N_{TD}}, \ \frac{6v_{DP}}{2N_{TD}}, \ \dots, \frac{2(N_{TD}-1)v_{DP}}{2N_{TD}} \ . \tag{6.42}$$

Using the distributed voltages of (6.42) to equate the electrostatic energy across all of the axial turn capacitances we obtain an equivalent value for the disc pair, denoted C_{EA} , i.e.

$$\frac{1}{2}C_{EA}v_{DP}^2 = \frac{1}{2}C_{AT}\left[\left[\frac{2v_{DP}}{2N_{TD}}\right]^2 + \left[\frac{4v_{DP}}{2N_{TD}}\right]^2 + \dots + \left[\frac{2(N_{TD}-1)v_{DP}}{2N_{TD}}\right]^2\right].$$
 (6.43)

From the mathematical identity,

$$1^{2} + 2^{2} + \dots + (n-1)^{2} = \frac{n(n-1)(2n-1)}{6} , \qquad (6.44)$$

equation (6.43) can be simplified to,

$$C_{EA} = \frac{(N_{TD} - 1)(2N_{TD} - 1)}{6N_{TD}}C_{AT} \quad . \tag{6.45}$$

A linear distribution for a voltage v_{DS} applied across a winding section of N_{DS} discs is shown in Figure 6.12. It can be observed that the potential difference across any disc pair is $\frac{2v_{DS}}{N_{DS}}$. The energy across all of the disc pair axial capacitances for this winding section must be equal to the energy across an equivalent axial capacitance for the section. That is,

$$\frac{1}{2}C_{EAX}v_{DS}^2 = \frac{1}{2}C_{EA}\left[\frac{2v_{DS}}{N_{DS}}\right]^2 \times (N_{DS} - 1) \quad , \tag{6.46}$$

where C_{EAX} is the equivalent axial capacitance for a winding section and $(N_{DS} - 1)$ is the number of disc pair axial capacitors in the winding section. Equation (6.46) can be



Figure 6.12: Voltage distribution across a winding section of N_{DS} discs

rewritten as,

$$C_{EAX} = \frac{4(N_{DS} - 1)}{N_{DS}^2} C_{EA} \quad . \tag{6.47}$$

A relationship in terms of the transformer dimensions of Figure 6.6 for (6.47) can be found by substituting (6.34) and (6.45) into (6.47),

$$C_{EAX} \approx \frac{4(N_{DS}-1)}{N_{DS}^2} \cdot \frac{(N_{TD}-1)(2N_{TD}-1)}{6N_{TD}} \cdot \frac{\epsilon\pi \left(\Theta_{HOD}^2 - \Theta_{HID}^2\right)}{4d_S (N_{TD}-1)} \approx \frac{\epsilon\pi (N_{DS}-1) \left(\Theta_{HOD}^2 - \Theta_{HID}^2\right)}{3d_S N_{DS}^2} , \qquad (6.48)$$

noting that $N_{TD} \gg 1$. This simplification cannot be assumed for N_{DS} (the number of discs per section) as it is linked to the modelling resolution and the disc pair lower limit, i.e. there must be at least two discs per section $(N_{DS} \ge 2)$.

The equivalent series capacitance of a continuous disc winding for winding section i is determined by summing the equivalent sectional radial capacitance (6.41) and the equivalent sectional axial capacitance (6.48),

$$C_{SXi} = \frac{\epsilon \pi w \left(\Theta_{HOD} + \Theta_{HID}\right)}{2t_p N_{DS} N_{TD}} + \frac{\epsilon \pi \left(N_{DS} - 1\right) \left(\Theta_{HOD}^2 - \Theta_{HID}^2\right)}{3 \mathrm{d}_S N_{DS}^2} \quad . \tag{6.49}$$

Interleaved Disc Winding Equivalent Series Capacitance

Interleaving is the process of separating sequential turns by a turn that is electrically more distant. An example of interleaving turns within a disc pair is shown in Figure 6.13. Assuming that the applied voltage, v_{DP} , is uniformly distributed across each turn,



Figure 6.13: Simplified model of an interleaved winding disc pair.

an increase in the separation distance of sequential turns results in an increase in the potential difference between physically adjacent turns. This has the effect of increasing the equivalent series capacitance [71].

There are a number of interleaving options. Here we derive the series capacitance for an interleaved winding of two strands over two discs [53]. Since the axial distance between each disc is significantly greater than the distance between two radially adjacent turns, the axial turn capacitance in an interleaved winding can be considered small relative to the radial capacitance component and is neglected.

With reference to the disc pair shown in Figure 6.13, it can be observed that there are 12 turns per disc pair, or $2N_{TD} = 12$ where N_{TD} is the number of turns per disc. Between each adjacent turn is a radial turn capacitance C_{RT} with a total of 10, or $2(N_{TD} - 1)$, capacitors per disc pair. There are 6, N_{TD} , adjacent turn pairs where there is a turn separation of 6, N_{TD} . These adjacent turn pairs are (1,7), (2,8), (3,9), (4,10), (5,11) and (6,12). There are 4, $(N_{TD} - 2)$, adjacent turn pairs where there is a turn separation of 5, $(N_{TD} - 1)$. These are (2,7), (3,8), (5,10) and (6,11). A voltage v_{DP} applied to the disc pair such that it is uniformly distributed across each turn will result in a voltage drop across each sequential turn of $\frac{v_{DP}}{2N_{TD}}$. The capacitors that have a 6, N_{TD} , turn separation will possess a voltage of $\frac{N_{TD}v_{DP}}{2N_{TD}}$, or $\frac{v_{DP}}{2}$. The capacitors that have a 5, $(N_{TD} - 1)$, turn separation will possess a voltage of $\frac{(N_{TD} - 1)v_{DP}}{2N_{TD}}$. An equivalent value of capacitance for the disc pair, C_{ER} , can be determined by equating its electrostatic energy to the electrostatic energy of all of the individual capacitances of the disc pair

[71],

$$\frac{1}{2}C_{ER}v_{DP}^2 = \frac{1}{2}C_{RT}\left[\frac{v_{DP}}{2}\right]^2 N_{TD} + \frac{1}{2}C_{RT}\left[\frac{(N_{TD}-1)v_{DP}}{2N_{TD}}\right]^2 (N_{TD}-2) \quad . \tag{6.50}$$

Rearranging (6.50) in terms of C_{ER} , we have

$$C_{ER} = \left[\frac{N_{TD}}{4} + (N_{TD} - 2)\left[\frac{(N_{TD} - 1)}{2N_{TD}}\right]^2\right]C_{RT} \quad .$$
(6.51)

Since $N_{TD} \gg 1$, (6.51) reduces to,

$$C_{ER} \approx \frac{N_{TD}C_{RT}}{2} \quad . \tag{6.52}$$

As (6.52) is based on a disc pair, the equivalent series capacitance for winding section i of N_{DS} discs for generic phase X, is given by,

$$C_{SXi} \approx \frac{N_{TD}C_{RT}}{N_{DS}} \quad . \tag{6.53}$$

The equivalent series capacitance for an interleaved winding section in terms of transformer dimensions in Figure 6.6, can be found by substituting (6.36) into (6.53),

$$C_{SXi} \approx \left(\frac{N_{TD}}{N_{DS}}\right) \left(\frac{\epsilon \pi w \left(\Theta_{HOD} + \Theta_{HID}\right)}{2t_p}\right) \\ \approx \frac{\epsilon \pi w N_{TD} \left(\Theta_{HOD} + \Theta_{HID}\right)}{2t_p N_{DS}} .$$
(6.54)

The interleaved disc winding will have a larger series capacitance than the continuous disc winding configuration, however it is more expensive to manufacture. As a result, the winding configuration used in a power transformer is generally dependent on a tradeoff between performance and manufacturing costs [52].

6.5.6 Dielectric Loss

In addition to the displacement current of a capacitor, dielectric material will also experience losses through conduction and material polarization [16]. The relationship between displacement and conduction currents for a capacitive element can be concisely described through analysis of Ampere's circuital law. In differential form, Ampere's circuital law [32] is given by,

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial \mathbf{t}} \quad , \tag{6.55}$$

where **J** is the conduction current density and $\frac{\partial \mathbf{D}}{\partial t}$ is the displacement current density. Since the field vectors are sinusoidal functions of time, (6.55) can be expressed in time-

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harmonic form as,

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{j}\omega\epsilon\mathbf{E} \quad , \tag{6.56}$$

where ϵ is the permittivity of the medium and ω the angular frequency. If there is conduction across the dielectric medium of a capacitor, the conductance must be greater than zero ($\sigma > 0$) hence conduction current density can be expressed as,

$$\mathbf{J} = \sigma \mathbf{E} \quad . \tag{6.57}$$

By combining (6.56) and (6.57),

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon) \mathbf{E}$$

= $j\omega \left(\epsilon + \frac{\sigma}{j\omega}\right) \mathbf{E}$
= $j\omega\epsilon_c \mathbf{E}$. (6.58)

The complex permittivity ϵ_c is defined as,

$$\epsilon_c = \epsilon + \frac{\sigma}{j\omega} = \epsilon' - j\epsilon'' \quad , \tag{6.59}$$

where,

$$\epsilon' = \epsilon \tag{6.60}$$

$$\epsilon^{''} = \frac{\delta}{\omega} . \tag{6.61}$$

Now the ratio of ϵ'' to ϵ' relates the magnitude of the conduction current to the displacement current. This ratio is the loss tangent of the capacitive medium and is a measure of the ohmic loss [32],

$$\tan \delta_c = \frac{\sigma}{\omega \epsilon} = \frac{\epsilon''}{\epsilon'} \quad , \tag{6.62}$$

where δ_c is referred to as the loss angle. Monitoring the variation in the loss tangent of a transformer's insulation system over time is a useful technique for gauging the thermal age of the insulation, and hence its remaining service life [23].

An ideal capacitor can be expressed in terms of permittivity and geometric topology. For example, a parallel plate capacitor constructed from two plates of area A, separated by a distance d and assumed to have no losses, will have a capacitance of,

$$C = \epsilon \frac{A}{d} = \epsilon \tilde{C} , \qquad (6.63)$$

where C is the ideal capacitance and \tilde{C} represents the geometric capacitance [16]. For

a non-ideal capacitor the conduction losses can be included by substituting the complex permittivity, ϵ_c of (6.59), for the ideal permittivity, ϵ in (6.63), giving the non-ideal capacitance⁵,

$$\mathfrak{C} = \epsilon_c \tilde{C} \quad . \tag{6.64}$$

From (6.59) and (6.64), the circuit admittance for a non-ideal capacitor is [16, 39],

$$Y = j\omega \mathfrak{C}$$

= $j\omega \epsilon_c \tilde{C}$
= $\omega \epsilon'' \tilde{C} + j\omega \epsilon' \tilde{C}$
= $G + j\omega C$. (6.65)

It can be observed from (6.65) that the admittance of a non-ideal capacitor can be represented as the parallel combination of a loss conductance G and an ideal capacitor [52]. Combining the loss conductance of (6.65) with (6.61),

$$G = \omega \epsilon'' \tilde{C}$$

= $\sigma \tilde{C}$. (6.66)

From (6.66) it is seen that the loss conductance of a capacitor is equivalent to the geometrically scaled conductance of the capacitor's dielectric. With reference to (6.62) and (6.63), (6.66) can be rewritten as,

$$G = \omega C \, \tan \delta_c \ . \tag{6.67}$$

The tan δ_c relationship is dependent upon the dielectric material. For example, for Nomex[®] paper insulation, tan δ_c can be modelled as [54],

$$\tan \delta_c = 0.07 \left(1 - \frac{6}{7} e^{-(0.05\omega \times 10^{-6})} \right) \quad , \tag{6.68}$$

and therefore the loss conductance G in this case is a non-linear frequency dependent term.

As discussed within Section 6.5, there are several dominant capacitance relationships which need to be considered when modelling a transformer. In each case the permittivity of the medium is a combination of materials, such as paper/pressboard in mineral oil as discussed in Section 2.6. Without a priori knowledge of the internal dimensions and material makeup of the transformer, it is necessary for the dielectric loss of each capacitance to be treated as independent parameters.

The circuit element used to represent each of the non-ideal capacitors used in the transformer model takes the form shown in Figure 6.14.

⁵The composite nature of the capacitance element is symbolised by the use of the Fraktur character



Figure 6.14: Circuit element for a non-ideal capacitor

6.6 Model for Generic Phase X

The model for generic phase X shown in Figure 6.15 is constructed from the circuit elements that have been defined in the previous sections. A generic three phase transformer model can then be constructed through the interconnection of three of the generic phase model blocks, nominally designated X, Y and Z. Figure 6.16 provides examples for Dyn1 and YNyn0 vector groups.

The high voltage winding of the generic phase model shown in Figure 6.15, is based on the series connection of alternating resistive, \mathfrak{R}_X , and inductive, \mathfrak{L}_X , circuit elements (Sections 6.3 and 6.4). Each $\mathfrak{R}_X \mathfrak{L}_X$ pair represents a section of the model's *n* high voltage winding sections. The low voltage winding's resistive and inductive circuit elements, \mathfrak{R}_x and \mathfrak{L}_x respectively, run in parallel with their high voltage winding equivalents.

For the distributed non-ideal capacitance that exists between the high and low voltage windings (Section 6.5.1), a capacitive element, \mathfrak{C}_{Xx} , is placed between each winding section in the model. In order to balance the capacitance distribution, the value of the capacitive element at the winding ends of each phase is $\frac{1}{2}\mathfrak{C}_{Xx}$.

The non-ideal capacitance between a low voltage winding section and the transformer core (Section 6.5.2), is represented by the capacitive element, \mathfrak{C}_{gx} . As with \mathfrak{C}_{Xx} , the capacitive element at the winding ends of each phase is $\frac{1}{2}\mathfrak{C}_{gx}$.

With reference to Figure 6.15, the capacitive element, \mathfrak{C}_{XY} , represents the non-ideal capacitance between the equivalent high voltage winding sections of phase X and phase Y (Section 6.5.3). Likewise, capacitive element \mathfrak{C}_{ZX} , represents the non-ideal capacitance between the equivalent high voltage winding sections of phase Z and phase X. The capacitive coupling between adjacent phase high voltage winding sections is repeated for generic phase models Y and Z as given in Table 6.2. In practice only phases A and B and phases B and C are physically adjacent. Therefore the capacitance that is representative of the interphase capacitance between phases A and C must approach zero. For example, if the generic phases X, Y and Z were nominally allocated to phases A, B and C respec-



Figure 6.15: Model for generic phase X.


Figure 6.16: Generic transformer model examples; (a) Dyn1, (b) YNyn0.

Generic Phase Model	Interphase Capacitive Elements
X	\mathfrak{C}_{XY} , \mathfrak{C}_{ZX}
Y	$\mathfrak{C}_{XY}\;,\mathfrak{C}_{YZ}$
Z	$\mathfrak{C}_{YZ}\;,\mathfrak{C}_{ZX}$

Table 6.2: Interphase capacitive elements linking each of the generic phase models.

tively, then \mathfrak{C}_{ZX} would be set to zero in the transformer model. Again to balance the interphase capacitance distribution, the capacitive element values at the winding ends of each phase are at half value, i.e. between phase X and Y, at the winding ends, the sectional capacitance is $\frac{1}{2}\mathfrak{C}_{XY}$.

The capacitive element, \mathfrak{C}_{gX} , represents the non-ideal capacitance between a high voltage winding section of phase X and the transformer tank wall. For phases Y and Z, the capacitive elements are designated \mathfrak{C}_{gY} and \mathfrak{C}_{gZ} . As discussed in Section 6.5.4, these components are also dependent upon the phase allocation of the transformer due to the geometric layout of the tank, i.e. in practice only phases A and C are adjacent to a transformer tank end wall. There can also be a significant difference between the phase A and phase C high voltage winding to end wall dimensions due to a variety of transformer construction strategies [53]. An example might be the location of the tap changer. However, due to the additional influence of the transformer tank end wall, it would be expected that the parameter estimate for the high voltage winding to tank capacitance for phases A and C would be larger than that of phase B,

$$C_{gA}, \ C_{gC} > C_{gB}$$
 . (6.69)

This parameter variation is demonstrated by example in Chapter 9. Once again the values of the capacitive elements at the winding ends of each phase are considered at half value, i.e. for phase X, $\frac{1}{2}\mathfrak{C}_{gX}$.

The series capacitive elements for the high and low voltage winding sections of generic phase X are \mathfrak{C}_{SX} and \mathfrak{C}_{Sx} respectively. The derivation of the series capacitance is based on the fact that when a sharp impulse is observed across a winding, it will behave like a string of capacitors placed in series [53]. As a result, the model's series capacitive elements are placed in parallel with their respective inductance.

Note that any bushing capacitances associated with phase X are incorporated into model capacitances C_{gX1} and $C_{gX(n+1)}$ for the HV terminals and C_{gx1} and $C_{gx(n+1)}$ for the LV terminals.

6.7 Conclusion

This chapter presents a wide-band frequency, generic phase, transformer model. It has used results from Chapters 3, 4 and 5 in order to develop the inductive element \mathfrak{L} of Section 6.3. The inductive element is a composite component that takes into account all of the associated inductive coupling between the various winding sections including both the ferromagnetic and leakage contributions. It is frequency, geometry and winding dependent.

The chapter then introduces a composite series resistance element \Re in Section 6.4. Along with the DC resistance losses, the circuit element incorporates frequency dependent skin and proximity effects. The section concludes by proposing a method for the determination of the composite resistance using estimates of the cross sectional area of the conductor and the number of winding layers. The advantage this approach has over a simple resistance estimate is that these two parameters can be tightly constrained. This is discussed further in Chapter 8.

Section 6.5 develops relationships for each of the major capacitive coupling influences that exist throughout the transformer structure. These include the capacitance between windings, windings and the core, windings of adjacent phases, windings and the tank wall and the series capacitance across a winding. Series capacitance relationships were derived for both continuous disc and interleaved disc windings. To take into account conduction losses associated with each capacitor, a loss conductance was placed in parallel, resulting in the non-ideal capacitive element \mathfrak{C} being defined.

The generic phase model was presented in Figure 6.15. This approach is ideal for asymmetric injection modelling where individual sources may be applied to various terminal connections, as is the case with FRA. Source injection within a winding can also be accommodated which is useful for partial discharge location techniques. The model is designed to function across a wide range of frequencies, nominally from DC to greater than 1MHz (Chapter 3). This bandwidth is suitable for FRA and PD location applications.

Chapter 7

Modelling for Frequency Response Analysis

7.1 Introduction

This chapter develops models for different FRA tests and transformer vector groups. A layered approach is adopted in order to partition the overall model into logical segments. The first layer represents the generic phase models X, Y and Z, the second layer is the connection of these three models to form the transformer vector group, and the third layer is the FRA test connection and associated input and output impedances. We specifically focus on three different types of FRA tests applied to both Dyn1 and Dyn11 connected transformers.

This chapter is structured in the following manner. Section 7.2 introduces the different FRA test connections and illustrates the three tests which are the focus for the remainder of the chapter. Section 7.3 discusses the layered modelling approach. Section 7.4 converts the complex models into a normal tree form to facilitate the development of a state space model. Section 7.5 derives transfer functions for each of the FRA tests. Section 7.6 provides an example which determines the transfer function between a terminal pair¹ for a particular FRA test connection. Concluding remarks are then given in Section 7.7.

7.2 Test Configurations for Frequency Response Analysis

The main FRA test types as classified by CIGRE are the End to End Open Circuit, End to End Short Circuit, Capacitive Interwinding, and the Inductive Interwinding test [6]. The End to End Open Circuit test injects a signal into one end of a winding and measures the response at the other end of the same winding. It is the most commonly used test due to its simplicity and its ability to examine individual windings separately. The End to End Short Circuit test is similar to the Open Circuit test, however with a winding

¹The term terminal pair refers to the two transformer terminals connected as the FRA input and output terminals during an FRA test. The remaining terminals are not connected to the test equipment.



Figure 7.1: FRA test configurations for a Dyn vector group; (a) High Voltage Winding End to End Open Circuit test, (b) Low Voltage Winding End to End Open Circuit test, (c) Capacitive Interwinding test.

of the same phase short circuited. This test removes the influence of the magnetising inductance such that the leakage inductance dominates the low frequency response. At high frequencies the response is similar to the End to End Open Circuit test [6]. The Capacitive Interwinding test injects a signal into one end of a winding and measures the response at the end of another winding of the same phase. As the test name suggests, this test is particularly sensitive to the interwinding capacitance that exists between windings of the same phase. The Inductive Interwinding test is connected as per the Capacitive Interwinding test, with the exception that the opposite ends of both windings are connected to ground.

An article by Satish et al. [98] classified the sensitivity of a broad range of FRA test connections. Sensitivity was quantified by counting the number of observed natural frequencies in each of the FRA tests, resulting in several categories. Of the standard FRA tests described above, Satish et al. considered the End to End Open Circuit test to be in the highest sensitivity category. The End to End Short Circuit, Capacitive Interwinding and Inductive Interwinding tests were each classified in the second tier of the sensitivity categories. In 2006 Jayasinghe et al. [61] conducted research on the sensitivity of different FRA measurement connections and their ability to detect different types of faults. In this article, the Capacitive Interwinding FRA test was found to be more sensitive to axial displacement and radial deformation than the End to End Open Circuit test. In addition, an article by Ryder et al. [97] in 2003 proposes the use of a Capacitive Interwinding FRA test as a suitable means of detecting the axial collapse of a 300MVA autotransformer winding. Based on these articles and due to the large number of FRA test and vector group permutations, we focus on the End to End Open Circuit and Capacitive Interwinding tests for Dyn connected transformers. We note that the modelling approach discussed in this chapter could equally be applied to all FRA test connection and vector group combinations. The FRA tests utilised here are shown in Figure 7.1. Each of these three FRA tests have three terminal permutations. This results in nine unique frequency $responses^2$ which can then be used to estimate the parameters for a transformer model.

7.2.1 High Voltage Winding End to End Open Circuit Test

Figure 7.1(a) shows the High Voltage Winding End to End Open Circuit test on a generic phase Dyn connected transformer. This test involves injecting a swept frequency signal into one of the high voltage terminals and recording the output response on another. All of the remaining terminals are left unconnected. Note that the test is repeated for all three terminal pair permutations.

To facilitate FRA modelling, the input terminal for this test sequence is labelled generic terminal X and the output terminal generic terminal Y (terminals Z, x, y and

 $^{^2 {\}rm The}$ frequency responses are also dependent upon the vector group, i.e. whether the vector group is a Dyn1 or Dyn11.



Figure 7.2: Vector group topologies; (a) Dyn1, (b) Dyn11.

z are unconnected). The modelling approach involves the substitution of the physical phases, A through C, into a prescribed generic counterpart, X through Z, for each of the three FRA test combinations (AB, BC and CA). The substitution of a physical phase into the generic phase includes all of the associated physical relationships related to that phase as discussed in Section 6.2.

The generic phase approach also facilitates modelling the phase order differences between Dyn1 and Dyn11 vector groups. With reference to Figure 7.2, it is noted that the high voltage winding of a Dyn1 vector group has phase sequence A-C-B whereas a Dyn11 has phase sequence A-B-C. By comparing the physical connections of Figure 7.2 with the generic phase connections of Figure 7.1(a), the generic phase allocation for each test is established (see Table 7.1).

We note with reference to Figure 7.1(a) that the winding whose terminals have a direct connection to the FRA input and output terminals, is labelled winding X. This is important due to the disparity in inductance between a phase B winding and that of phases A or C. Due to this disparity, the low frequency response is dependent upon the FRA test pair and the transformer's vector group. This is discussed in detail in Appendix A.

7.2.2 Low Voltage Winding End to End Open Circuit Test

The Low Voltage Winding End to End Open Circuit test on a generic phase Dyn connected transformer is shown in Figure 7.1(b). This test injects a swept frequency signal into a low voltage terminal and records the output response at the neutral. All other terminals are left unconnected. The test is performed sequentially between each of the low voltage terminals and neutral.

Again, to facilitate modelling this FRA test, the low voltage generic terminal x is the source input and the output is the low voltage winding neutral. The modelling approach involves the substitution of the physical phases into their prescribed generic counterparts

Vector GroupFRA Test $(v_{IN}v_{OUT})$		X/x	Y/y	\mathbf{Z}/\mathbf{z}
	AC	A/a	C/c	B/b
Dyn1	ВА	B/b	A/a	$\mathrm{C/c}$
	CB	$\mathrm{C/c}$	B/b	A/a
	AB	A/a	B/b	C/c
Dyn11	BC	B/b	$\mathrm{C/c}$	A/a
	CA	C/c	A/a	B/b

Table 7.1: Generic phase allocation for FRA tests of Dyn1 and Dyn11 vector groups for High Voltage Winding End to End Open Circuit tests.

Table 7.2: Generic phase allocation for FRA tests of Dyn1 and Dyn11 vector groups for Low Voltage Winding End to End Open Circuit tests.

Vector Group	FRA Test $(v_{IN}v_{OUT})$	X/x	Y/y	\mathbf{Z}/\mathbf{z}
	an	A/a	C/c	B/b
Dyn1	bn	B/b	A/a	$\mathrm{C/c}$
	cn	C/c	$\rm B/b$	A/a
	an	A/a	B/b	$\mathrm{C/c}$
Dyn11	bn	B/b	$\mathrm{C/c}$	A/a
	cn	C/c	A/a	B/b

for each of the three FRA tests (an, bn, and cn). The FRA tests and their corresponding generic phase allocation for the Low Voltage Winding End to End Open Circuit test is shown in Table 7.2.

7.2.3 Capacitive Interwinding Test

Figure 7.1(c) shows the Capacitive Interwinding test on a generic phase Dyn connected transformer. This test is conducted between the high and low voltage terminals of a given phase, with the remaining terminals left unconnected. The test is then repeated for the other two phases.

To facilitate modelling of this FRA test, the injection point is the generic high voltage terminal X and the output is the generic low voltage terminal x. The generic phase allocation for each of the three FRA tests (Aa, Bb and Cc) is shown in Table 7.3.

Vector Group	FRA Test $(v_{IN}v_{OUT})$	X/x	Y/y	\mathbf{Z}/\mathbf{z}
	Aa	A/a	C/c	B/b
Dyn1	Bb	B/b	A/a	C/c
	Cc	C/c	B/b	A/a
	Aa	A/a	B/b	C/c
Dyn11	Bb	B/b	$\mathrm{C/c}$	A/a
	Cc	C/c	A/a	B/b

Table 7.3: Generic phase allocation for FRA tests of Dyn1 and Dyn11 vector groups for Capacitive Interwinding tests.



Figure 7.3: Model layers.

7.3 Model Layers

A comprehensive system model of a three phase transformer undergoing an FRA test is constructed by connecting three instances of the generic phase model developed in Chapter 6, together with the required FRA test connections. To facilitate modelling each of the vector group and FRA test types, it is convenient to consider the system model in terms of layers. The first layer is the three instances (X, Y and Z) of the generic phase model. The second and third layers are the vector group of the transformer and the FRA test that is being prescribed respectively. The layer analogy is shown in Figure 7.3. A mathematical model is now developed for each of the FRA tests discussed in Section 7.2.

7.4 Conversion of Models to Normal Tree Form

To analyse the complex circuits in the model structures that are proposed, it is mathematically advantageous to first convert the proposed model(s) into *normal tree* form [95]. This conversion process requires the application of a strict set of rules.

By definition [95], a normal tree can only have one path, via branches, between any pair of nodes. In addition, a normal tree must have all independent voltage sources and the maximum possible number of capacitances as tree branches. The remaining circuit elements which were not allocated as branches are classified as links. To obtain the normal tree form, tree links must contain the maximum possible number of inductances [95].

To satisfy these rules with respect to the proposed model, a few adjustments to the model are necessary. The first adjustment is the decomposition of the capacitive elements into their respective components. The second adjustment is for the sake of mathematical convenience in later analysis and requires the conversion of all parallel conductances to resistances and all series resistances to conductances.

In the normal tree, a capacitive element's leakage conductance G_{Xi} (as discussed in Section 6.5.6), is represented by the leakage resistance R_{Xi} where,

$$R_{Xi} = \frac{1}{G_{Xi}} {.} {(7.1)}$$

For the winding series resistance element $\Re_{Xi}(\omega)$ (as discussed in Section 6.4.3), its normal tree representation is conductance,

$$\mathfrak{G}_{Xi}(\omega) = \frac{1}{\mathfrak{R}_{Xi}(\omega)} \quad . \tag{7.2}$$

These changes are also applied to phases Y and Z for both the HV and LV parameters.

Through application of these rules, the normal tree associated with each of the FRA tests is obtained in the following manner. Step one, allocate the FRA injection voltage source as a branch. For the High Voltage End to End Test and the Capacitive Interwinding test, the source is located between the high voltage terminal X and earth. For the Low Voltage End to End FRA test, the source is between the low voltage terminal x and earth. The next step is the allocation of branches to as many of the capacitors as possible (whilst maintaining the single path between nodes rule). After this step, the only unconnected nodes that remain are the nodes between the winding inductive elements \mathfrak{L} and the conductive elements \mathfrak{G} , for both the HV and LV windings. Since one of the normal tree rules was to maximise the number of tree links associated with inductances, the final tree branches need to be allocated to the winding conductive elements, \mathfrak{G} . All of the remaining circuit elements should now be replaced by tree links. This last step also includes the FRA termination resistor, R_T , whose location is FRA test dependent. The normal tree representations of the three FRA test system models are shown in Figures 7.4, 7.5 and 7.6.



Figure 7.4: Normal tree representation for a High Voltage Winding End to End Open Circuit FRA test on a generic phase Dyn connected model.



Figure 7.5: Normal tree representation for a Low Voltage Winding End to End Open Circuit FRA test on a generic phase Dyn connected model.



Figure 7.6: Normal tree representation for a Capacitive Interwinding FRA test on a generic phase Dyn connected model.

7.5 Development of the Mathematical Model

For system analysis it is convenient to use a state space representation. This section develops state space equations for application to any of the FRA test system models discussed in the previous section. The derivations used to develop the state space representation are based on the work by Rohrer [95].

7.5.1 Branch to Link Coupling Matrix

In a lumped network topology such as in Figures 7.4, 7.5 and 7.6, the branch voltages, $\mathbf{v}_b(t)$, and branch currents, $\mathbf{i}_b(t)$, can be partitioned with respect to their component type, i.e.

$$\mathbf{v}_{b}(t) = \begin{bmatrix} \mathbf{v}_{V}(t) \\ \mathbf{v}_{C}(t) \\ \mathbf{v}_{G}(t) \\ \mathbf{v}_{\mathfrak{S}}(t) \\ \mathbf{v}_{S}(t) \\ \mathbf{v}_{R}(t) \\ \mathbf{v}_{\mathfrak{L}}(t) \end{bmatrix}$$
(7.3)
$$\mathbf{i}_{b}(t) = \begin{bmatrix} \mathbf{i}_{V}(t) \\ \mathbf{i}_{C}(t) \\ \mathbf{i}_{\mathcal{G}}(t) \\ \mathbf{i}_{\mathcal{G}}(t) \\ \mathbf{i}_{\mathcal{S}}(t) \\ \mathbf{i}_{\mathfrak{L}}(t) \\ \mathbf{i}_{\mathfrak{L}}(t) \end{bmatrix} , \qquad (7.4)$$

where $\mathbf{v}_b(t)$ and $\mathbf{i}_b(t)$ are both vectors and the subscript V denotes an independent voltage source, C capacitance, \mathfrak{G} conductance, S elastance (reciprocal capacitance for capacitor links), R resistance and \mathfrak{L} inductance³. A fundamental cutset occurs when the removal of one tree branch and a minimum number of links separates the network tree into two parts [95]. Hence, a fundamental cutset is just a graphical representation of Kirchoff's current law. On this basis, the fundamental cutset equations for a normal tree can be defined as,

$$\mathbf{Qi}_b(t) \triangleq \mathbf{0} \quad , \tag{7.5}$$

 $^{^{3}}$ Note that the conductance and inductance are both composite elements (as discussed in Chapter 6) and are symbolised as such by Fraktur letters.

where $\mathbf{0}$ is a zero vector of appropriate dimensions and,

$$\mathbf{Q} \triangleq \left[\mathbf{I}:\mathbf{F}\right] \quad . \tag{7.6}$$

In (7.6), \mathbf{I} is the identity matrix and \mathbf{F} is a matrix that relates the network tree branch to link coupling. By grouping the respective branch to link connectivity relationships of each of the circuit elements, \mathbf{F} can be defined as,

$$\mathbf{F} = \begin{vmatrix} \mathbf{F}_{VS} & \mathbf{F}_{VR} & \mathbf{F}_{V\mathcal{L}} \\ \mathbf{F}_{CS} & \mathbf{F}_{CR} & \mathbf{F}_{C\mathcal{L}} \\ \mathbf{0} & \mathbf{F}_{\mathfrak{GR}} & \mathbf{F}_{\mathfrak{GL}} \end{vmatrix} , \qquad (7.7)$$

where $\mathbf{0}$ is a zero submatrix of appropriate dimensions and,

\mathbf{F}_{VS}	=	voltage branch to elastance link submatrix,
\mathbf{F}_{VR}	=	$voltage \ branch \ to \ resistance \ link \ submatrix,$
$\mathbf{F}_{V\mathfrak{L}}$	=	$voltage \ branch \ to \ inductance \ link \ submatrix,$
\mathbf{F}_{CS}	=	${\rm capacitance}\ {\rm branch}\ {\rm to}\ {\rm elastance}\ {\rm link}\ {\rm submatrix},$
\mathbf{F}_{CR}	=	${\rm capacitance}{\rm branch}{\rm to}{\rm resistance}{\rm link}{\rm submatrix},$
$\mathbf{F}_{C\mathfrak{L}}$	=	capacitance branch to inductance link submatrix,
$\mathbf{F}_{\mathfrak{G}R}$	=	${\rm conductance\ branch\ to\ resistance\ link\ submatrix,}$
$\mathbf{F}_{\mathfrak{GL}}$	=	${\rm conductance}\ {\rm branch}\ {\rm to}\ {\rm inductance}\ {\rm link}\ {\rm submatrix}.$

There is no $\mathbf{F}_{\mathfrak{G}S}$ as, by definition, tree branch allocation is given a higher priority to capacitances relative to conductances. As a result $\mathbf{F}_{\mathfrak{G}S}$ is a zero submatrix of the appropriate dimension.

7.5.2 Application of Kirchoff's Laws

From (7.5), (7.6) and (7.7), the fundamental cutset, or Kirchoff's current law, equations can be written as,

$$\mathbf{i}_{V}(t) + \mathbf{F}_{VS}\mathbf{i}_{S}(t) + \mathbf{F}_{VR}\mathbf{i}_{R}(t) + \mathbf{F}_{V\mathfrak{L}}\mathbf{i}_{\mathfrak{L}}(t) = \mathbf{0}$$
(7.8)

$$\mathbf{i}_C(t) + \mathbf{F}_{CS}\mathbf{i}_S(t) + \mathbf{F}_{CR}\mathbf{i}_R(t) + \mathbf{F}_{C\mathfrak{L}}\mathbf{i}_{\mathfrak{L}}(t) = \mathbf{0}$$
(7.9)

$$\mathbf{i}_{\mathfrak{G}}(t) + \mathbf{F}_{\mathfrak{G}R}\mathbf{i}_{R}(t) + \mathbf{F}_{\mathfrak{G}\mathfrak{L}}\mathbf{i}_{\mathfrak{L}}(t) = \mathbf{0} . \qquad (7.10)$$

A fundamental loop is defined as being a loop of the network tree consisting of only one link with the remaining sections consisting of tree branches. A fundamental loop can be visualised as a graphical representation of Kirchoff's voltage law. For the networks under consideration, the fundamental loop equations can be represented by,

$$\mathbf{B}v_b(t) \triangleq \mathbf{0} \quad (7.11)$$

where,

$$\mathbf{B} \triangleq \left[-\mathbf{F}^{\mathbf{T}} : \mathbf{I} \right] \quad . \tag{7.12}$$

From (7.7), (7.11) and (7.12) the fundamental loop, or Kirchoff's voltage law, equations can be written as,

$$\mathbf{v}_{S}(t) - \mathbf{F}_{VS}^{T} \mathbf{v}_{V}(t) - \mathbf{F}_{CS}^{T} \mathbf{v}_{C}(t) = \mathbf{0}$$
(7.13)

$$\mathbf{v}_{R}(t) - \mathbf{F}_{VR}^{T} \mathbf{v}_{V}(t) - \mathbf{F}_{CR}^{T} \mathbf{v}_{C}(t) - \mathbf{F}_{\mathfrak{G}R}^{T} \mathbf{v}_{\mathfrak{G}}(t) = \mathbf{0}$$
(7.14)

$$\mathbf{v}_{\mathfrak{L}}(t) - \mathbf{F}_{V\mathfrak{L}}^{T} \mathbf{v}_{V}(t) - \mathbf{F}_{C\mathfrak{L}}^{T} \mathbf{v}_{C}(t) - \mathbf{F}_{\mathfrak{G}\mathfrak{L}}^{T} \mathbf{v}_{\mathfrak{G}}(t) = \mathbf{0} .$$
(7.15)

7.5.3 Differential Equations for the Capacitive Components

The capacitance branch relationships are defined as,

$$\begin{bmatrix} \mathbf{i}_C(t) \\ \mathbf{i}_S(t) \end{bmatrix} = \frac{d}{dt} \left\{ \begin{bmatrix} \mathbf{C}_C & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_S \end{bmatrix} \begin{bmatrix} \mathbf{v}_C(t) \\ \mathbf{v}_S(t) \end{bmatrix} \right\} , \qquad (7.16)$$

where \mathbf{C}_C is the capacitance branch parameter matrix and \mathbf{C}_S is the elastance link parameter matrix (in terms of capacitance). Both are positive definite diagonal matrices.

The capacitance cutset relationship of (7.9) can be rearranged,

$$\mathbf{i}_C(t) + \mathbf{F}_{CS}\mathbf{i}_S(t) = -\mathbf{F}_{CR}\mathbf{i}_R(t) - \mathbf{F}_{C\mathfrak{L}}\mathbf{i}_{\mathfrak{L}}(t) \quad .$$
(7.17)

The elastance fundamental loop of (7.13) can be written as,

$$\mathbf{v}_S(t) = \mathbf{F}_{CS}^T \mathbf{v}_C(t) + \mathbf{F}_{VS}^T \mathbf{v}_V(t) \quad .$$
(7.18)

Rearranging (7.18) we obtain,

$$\begin{bmatrix} \mathbf{v}_C(t) \\ \mathbf{v}_S(t) \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{F}_{CS}^T \end{bmatrix} \mathbf{v}_C(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{F}_{VS}^T \end{bmatrix} \mathbf{v}_V(t) .$$
(7.19)

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Utilising (7.16) and (7.19), the LHS of (7.17) can be expressed as,

$$\mathbf{i}_{C}(t) + \mathbf{F}_{CS}\mathbf{i}_{S}(t) = \begin{bmatrix} \mathbf{I} & \mathbf{F}_{CS} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{C}(t) \\ \mathbf{i}_{S}(t) \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{I} & \mathbf{F}_{CS} \end{bmatrix} \frac{d}{dt} \left\{ \begin{bmatrix} \mathbf{C}_{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{S} \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{F}_{CS}^{T} \end{bmatrix} \mathbf{v}_{C}(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{F}_{VS}^{T} \end{bmatrix} \mathbf{v}_{V}(t) \end{bmatrix} \right\}$$
$$= \frac{d}{dt} \left\{ (\mathbf{C}_{C} + \mathbf{F}_{CS}\mathbf{C}_{S}\mathbf{F}_{CS}^{T}) \mathbf{v}_{C}(t) + \mathbf{F}_{CS}\mathbf{C}_{S}\mathbf{F}_{VS}^{T}\mathbf{v}_{V}(t) \right\}.$$
(7.20)

Equating (7.20) with the RHS of (7.17) we have,

$$\frac{d}{dt}\left\{ \left(\mathbf{C}_C + \mathbf{F}_{CS} \mathbf{C}_S \mathbf{F}_{CS}^T \right) \mathbf{v}_C(t) + \mathbf{F}_{CS} \mathbf{C}_S \mathbf{F}_{VS}^T \mathbf{v}_V(t) \right\} = -\mathbf{F}_{CR} \mathbf{i}_R(t) - \mathbf{F}_{C\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t) \quad . \quad (7.21)$$

By defining \mathbb{C} such that,

$$\mathbb{C} \triangleq \mathbf{C}_C + \mathbf{F}_{CS} \mathbf{C}_S \mathbf{F}_{CS}^T \quad , \tag{7.22}$$

(7.21) can be written as,

$$\frac{d}{dt} \left\{ \mathbb{C} \mathbf{v}_C(t) + \mathbf{F}_{CS} \mathbf{C}_S \mathbf{F}_{VS}^T \mathbf{v}_V(t) \right\} = -\mathbf{F}_{CR} \mathbf{i}_R(t) - \mathbf{F}_{C\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t) \quad , \tag{7.23}$$

which defines the differential equations for the capacitive components.

7.5.4 Differential Equations for the Inductive Components

The inductance voltage-current relationship is defined as,

$$\mathbf{v}_{\mathfrak{L}}(t) = \frac{d}{dt} \left\{ \mathfrak{L}_M \mathbf{i}_{\mathfrak{L}}(t) \right\} \quad , \tag{7.24}$$

where \mathfrak{L}_M is the matrix for the tree link inductance parameters. Substituting (7.24) into the fundamental loop equation (7.15), we obtain the differential equations for the inductive components,

$$\frac{d}{dt} \left\{ \mathbf{\mathfrak{L}}_{M} \mathbf{i}_{\mathfrak{L}}(t) \right\} = \mathbf{F}_{V\mathfrak{L}}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{C\mathfrak{L}}^{T} \mathbf{v}_{C}(t) + \mathbf{F}_{\mathfrak{G}\mathfrak{L}}^{T} \mathbf{v}_{\mathfrak{G}}(t) \quad .$$
(7.25)

7.5.5 Conditioning the Differential Equations

To obtain a state variable model, it is necessary to eliminate $\mathbf{i}_R(t)$ and $\mathbf{v}_{\mathfrak{G}}(t)$ from equations (7.23) and (7.25). The tree branch conductance current-voltage relationship is given by,

$$\mathbf{i}_{\mathfrak{G}}(t) = \mathfrak{G}_M \mathbf{v}_{\mathfrak{G}}(t) \quad , \tag{7.26}$$

where \mathfrak{G}_M is the positive definite diagonal matrix for the conductance branch parameters. Substituting (7.26) into the cutset equation (7.10) and solving for $\mathbf{v}_{\mathfrak{G}}(t)$,

$$\mathfrak{G}_{M}\mathbf{v}_{\mathfrak{G}}(t) + \mathbf{F}_{\mathfrak{G}R}\mathbf{i}_{R}(t) + \mathbf{F}_{\mathfrak{G}\mathfrak{L}}\mathbf{i}_{\mathfrak{L}}(t) = \mathbf{0} ,$$

$$\therefore \mathbf{v}_{\mathfrak{G}}(t) = -\mathfrak{G}_{M}^{-1} \left[\mathbf{F}_{\mathfrak{G}R}\mathbf{i}_{R}(t) + \mathbf{F}_{\mathfrak{G}\mathfrak{L}}\mathbf{i}_{\mathfrak{L}}(t)\right] . \quad (7.27)$$

The voltage to current relationship for the network tree's link resistors is given by,

$$\mathbf{v}_R(t) = \mathbf{R}_R \mathbf{i}_R(t) \quad , \tag{7.28}$$

where \mathbf{R}_R is the positive definite diagonal matrix for the tree link resistances. Substituting (7.28) into the fundamental loop equation of (7.14), gives,

$$\mathbf{R}_{R}\mathbf{i}_{R}(t) - \mathbf{F}_{VR}^{T}\mathbf{v}_{V}(t) - \mathbf{F}_{CR}^{T}\mathbf{v}_{C}(t) - \mathbf{F}_{\mathfrak{G}R}^{T}\mathbf{v}_{\mathfrak{G}}(t) = \mathbf{0} .$$
(7.29)

Solving for $\mathbf{i}_R(t)$,

$$\mathbf{i}_{R}(t) = \mathbf{R}_{R}^{-1} \left[\mathbf{F}_{VR}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{CR}^{T} \mathbf{v}_{C}(t) + \mathbf{F}_{\mathfrak{G}R}^{T} \mathbf{v}_{\mathfrak{G}}(t) \right] \quad .$$
(7.30)

 $\mathbf{v}_{\mathfrak{G}}(t)$ can be removed from (7.30) through the substitution of (7.27),

$$\mathbf{i}_{R}(t) = \mathbf{R}_{R}^{-1} \left[\mathbf{F}_{VR}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{CR}^{T} \mathbf{v}_{C}(t) + \mathbf{F}_{\mathfrak{GR}}^{T} \mathbf{v}_{C}(t) + \mathbf{F}_{\mathfrak{GR}}^{T} \left(-\mathfrak{G}_{M}^{-1} \left[\mathbf{F}_{\mathfrak{GR}} \mathbf{i}_{R}(t) + \mathbf{F}_{\mathfrak{GL}} \mathbf{i}_{\mathfrak{L}}(t) \right] \right) \right]$$

$$\left[1 + \mathbf{R}_{R}^{-1} \mathbf{F}_{\mathfrak{G}R}^{T} \mathfrak{G}_{M}^{-1} \mathbf{F}_{\mathfrak{G}R}\right] \mathbf{i}_{R}(t) = \mathbf{R}_{R}^{-1} \left[\mathbf{F}_{VR}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{CR}^{T} \mathbf{v}_{C}(t) - \mathbf{F}_{\mathfrak{G}R}^{T} \mathfrak{G}_{M}^{-1} \mathbf{F}_{\mathfrak{G}\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t)\right]$$

$$\mathbf{R}_{R}^{-1}\left[\mathbf{R}_{R}+\mathbf{F}_{\mathfrak{G}R}^{T}\mathfrak{G}_{M}^{-1}\mathbf{F}_{\mathfrak{G}R}\right]\mathbf{i}_{R}(t) = \mathbf{R}_{R}^{-1}\left[\mathbf{F}_{VR}^{T}\mathbf{v}_{V}(t)+\mathbf{F}_{CR}^{T}\mathbf{v}_{C}(t)-\mathbf{F}_{\mathfrak{G}R}^{T}\mathfrak{G}_{M}^{-1}\mathbf{F}_{\mathfrak{G}\mathfrak{L}}\mathbf{i}_{\mathfrak{L}}(t)\right]$$

$$\therefore \mathbf{i}_{R}(t) = \left[\mathbf{R}_{R} + \mathbf{F}_{\mathfrak{G}R}^{T} \mathfrak{G}_{M}^{-1} \mathbf{F}_{\mathfrak{G}R}\right]^{-1} \\ \left[\mathbf{F}_{VR}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{CR}^{T} \mathbf{v}_{C}(t) - \mathbf{F}_{\mathfrak{G}R}^{T} \mathfrak{G}_{M}^{-1} \mathbf{F}_{\mathfrak{G}\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t)\right] .$$
(7.31)

Defining,

$$\mathbb{R} \triangleq \left[\mathbf{R}_R + \mathbf{F}_{\mathfrak{G}R}^T \mathfrak{G}_M^{-1} \mathbf{F}_{\mathfrak{G}R} \right] \quad , \tag{7.32}$$

(7.31) can be simplified to,

$$\mathbf{i}_{R}(t) = \mathbb{R}^{-1} \left[\mathbf{F}_{VR}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{CR}^{T} \mathbf{v}_{C}(t) - \mathbf{F}_{\mathfrak{G}R}^{T} \mathfrak{G}_{M}^{-1} \mathbf{F}_{\mathfrak{G}\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t) \right]$$
(7.33)

Similarly, it can be shown that $\mathbf{i}_R(t)$ can be removed from (7.27) through the substitution of (7.30). This results in,

$$\mathbf{v}_{\mathfrak{G}}(t) = -\mathbb{G}^{-1}\left[\mathbf{F}_{\mathfrak{G}R}\mathbf{R}_{R}^{-1}\mathbf{F}_{VR}^{T}\mathbf{v}_{V}(t) + \mathbf{F}_{\mathfrak{G}R}\mathbf{R}_{R}^{-1}\mathbf{F}_{CR}^{T}\mathbf{v}_{C}(t) + \mathbf{F}_{\mathfrak{G}\mathfrak{L}}\mathbf{i}_{\mathfrak{L}}(t)\right] , \qquad (7.34)$$

where,

$$\mathbb{G} \triangleq \mathfrak{G}_M + \mathbf{F}_{\mathfrak{G}R} \mathbf{R}_R^{-1} \mathbf{F}_{\mathfrak{G}R}^T .$$
(7.35)

The term $\mathbf{i}_R(t)$ can now be removed from the differential equation of (7.23) by the substitution of (7.33),

$$\frac{d}{dt} \left\{ \mathbb{C} \mathbf{v}_{C}(t) + \mathbf{F}_{CS} \mathbf{C}_{S} \mathbf{F}_{VS}^{T} \mathbf{v}_{V}(t) \right\} = -\mathbf{F}_{CR} \mathbb{R}^{-1} \left[\mathbf{F}_{VR}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{CR}^{T} \mathbf{v}_{C}(t) - \mathbf{F}_{\mathfrak{G}R}^{T} \mathfrak{G}_{M}^{-1} \mathbf{F}_{\mathfrak{G}\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t) \right] - \mathbf{F}_{C\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t) \quad . \quad (7.36)$$

Similarly, the term $\mathbf{v}_{\mathfrak{G}}(t)$ can be removed from the differential equation of (7.25) by the substitution of (7.34), i.e.

$$\frac{d}{dt} \left\{ \mathbf{\mathfrak{L}}_{M} \mathbf{i}_{\mathfrak{L}}(t) \right\} = -\mathbf{F}_{\mathfrak{G}\mathfrak{L}}^{T} \mathbb{G}^{-1} \left[\mathbf{F}_{\mathfrak{G}R} \mathbf{R}_{R}^{-1} \mathbf{F}_{VR}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{\mathfrak{G}R} \mathbf{R}_{R}^{-1} \mathbf{F}_{CR}^{T} \mathbf{v}_{C}(t) + \mathbf{F}_{\mathfrak{G}\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t) \right] + \mathbf{F}_{V\mathfrak{L}}^{T} \mathbf{v}_{V}(t) + \mathbf{F}_{C\mathfrak{L}}^{T} \mathbf{v}_{C}(t) \quad . \quad (7.37)$$

To simplify the differential equations (7.36) and (7.37), a set of matrices are now defined,

$$\mathbb{K}_{V\mathfrak{L}} \triangleq \mathbf{F}_{\mathfrak{G}\mathfrak{L}}^T \mathbb{G}^{-1} \mathbf{F}_{\mathfrak{G}R} \mathbf{R}_R^{-1} \mathbf{F}_{VR}^T - \mathbf{F}_{V\mathfrak{L}}^T , \qquad (7.38)$$

$$\mathbb{K}_{CC} \triangleq \mathbf{F}_{CR} \mathbb{R}^{-1} \mathbf{F}_{CR}^T , \qquad (7.39)$$

$$\mathbb{K}_{C\mathfrak{L}} \triangleq \mathbf{F}_{C\mathfrak{L}} - \mathbf{F}_{CR} \mathbb{R}^{-1} \mathbf{F}_{\mathfrak{G}R}^T \mathfrak{G}_M^{-1} \mathbf{F}_{\mathfrak{G}\mathfrak{L}} , \qquad (7.40)$$

$$\mathbb{K}_{CV} \triangleq \mathbf{F}_{CR} \mathbb{R}^{-1} \mathbf{F}_{VR}^T , \qquad (7.41)$$

$$\mathbb{K}_{\mathcal{L}C} \triangleq \mathbf{F}_{\mathcal{G}\mathcal{L}}^T \mathbb{G}^{-1} \mathbf{F}_{\mathcal{G}R} \mathbf{R}_R^{-1} \mathbf{F}_{CR}^T - \mathbf{F}_{C\mathcal{L}}^T , \qquad (7.42)$$

$$\mathbb{K}_{\mathfrak{L}\mathfrak{L}} \triangleq \mathbf{F}_{\mathfrak{G}\mathfrak{L}}^T \mathbb{G}^{-1} \mathbf{F}_{\mathfrak{G}\mathfrak{L}} . \tag{7.43}$$

Substituting (7.38) - (7.43) into (7.36) and (7.37) we have,

$$\frac{d}{dt} \left\{ \mathbb{C} \mathbf{v}_{C}(t) + \mathbf{F}_{CS} \mathbf{C}_{S} \mathbf{F}_{VS}^{T} \mathbf{v}_{V}(t) \right\} = -\mathbb{K}_{CV} \mathbf{v}_{V}(t) - \mathbb{K}_{CC} \mathbf{v}_{C}(t) - \mathbb{K}_{C\mathcal{L}} \mathbf{i}_{\mathfrak{L}}(t) \qquad (7.44)$$

and
$$\frac{d}{dt} \left\{ \mathfrak{L}_{M} \mathbf{i}_{\mathfrak{L}}(t) \right\} = -\mathbb{K}_{V\mathcal{L}} \mathbf{v}_{V}(t) - \mathbb{K}_{\mathfrak{LC}} \mathbf{v}_{C}(t) - \mathbb{K}_{\mathfrak{L}\mathfrak{L}} \mathbf{i}_{\mathfrak{L}}(t) \quad . \qquad (7.45)$$

7.5.6 State Space Model

To form the state space equations, it is convenient to define a new state variable, $\boldsymbol{q}(t)$ where,

$$\boldsymbol{q}(t) = \mathbb{C} \mathbf{v}_C(t) + \mathbf{F}_{CS} \mathbf{C}_S \mathbf{F}_{VS}^T \mathbf{v}_V(t) \quad , \tag{7.46}$$

which represents the "net charge per capacitance tree branch cutset". To facilitate the removal of $\mathbf{v}_C(t)$ from the differential equations of (7.44) and (7.45), (7.46) is rearranged in terms of $\mathbf{v}_C(t)$,

$$\mathbf{v}_C(t) = \mathbb{C}^{-1} \boldsymbol{q}(t) - \mathbb{C}^{-1} \mathbf{F}_{CS} \mathbf{C}_S \mathbf{F}_{VS}^T \mathbf{v}_V(t) \quad .$$
(7.47)

Another state variable $\phi(t)$ can be defined as,

$$\boldsymbol{\phi}(t) = \boldsymbol{\mathfrak{L}}_M \mathbf{i}_{\boldsymbol{\mathfrak{L}}}(t) \quad , \tag{7.48}$$

which represents the "net flux per inductance link fundamental loop". Rearranging (7.48) in terms of $\mathbf{i}_{\mathfrak{L}}(t)$,

$$\mathbf{i}_{\mathfrak{L}}(t) = \mathfrak{L}_M^{-1} \boldsymbol{\phi}(t) \quad . \tag{7.49}$$

The State Space equations are found by substituting (7.46), (7.47), (7.48) and (7.49) into the differential equations (7.44) and (7.45),

$$\frac{d}{dt} \{ \boldsymbol{q}(t) \} = - \mathbb{K}_{CC} \mathbb{C}^{-1} \boldsymbol{q}(t) - \mathbb{K}_{C\mathfrak{L}} \mathfrak{L}_{M}^{-1} \boldsymbol{\phi}(t)
+ \left[\mathbb{K}_{CC} \mathbb{C}^{-1} \mathbf{F}_{CS} \mathbf{C}_{S} \mathbf{F}_{VS}^{T} - \mathbb{K}_{CV} \right] \mathbf{v}_{V}(t) ,$$
(7.50)

$$\frac{d}{dt} \{ \boldsymbol{\phi}(t) \} = - \mathbb{K}_{\mathcal{L}C} \mathbb{C}^{-1} \boldsymbol{q}(t) - \mathbb{K}_{\mathcal{L}\mathcal{L}} \mathcal{L}_M^{-1} \boldsymbol{\phi}(t)
+ \left[\mathbb{K}_{\mathcal{L}C} \mathbb{C}^{-1} \mathbf{F}_{CS} \mathbf{C}_S \mathbf{F}_{VS}^T - \mathbb{K}_{V\mathcal{L}} \right] \mathbf{v}_V(t) .$$
(7.51)

Expressing the differential equations (7.50) and (7.51) in state space form,

$$\begin{bmatrix} \dot{\boldsymbol{q}}(t) \\ \dot{\boldsymbol{\phi}}(t) \end{bmatrix} = \mathbf{A} \begin{bmatrix} \boldsymbol{q}(t) \\ \boldsymbol{\phi}(t) \end{bmatrix} + \mathbf{B} \mathbf{v}_V(t) \quad , \tag{7.52}$$

where

$$\mathbf{A} = -\begin{bmatrix} \mathbb{K}_{CC} \mathbb{C}^{-1} & \mathbb{K}_{C\mathfrak{L}} \mathfrak{L}_{M}^{-1} \\ \mathbb{K}_{\mathfrak{L}C} \mathbb{C}^{-1} & \mathbb{K}_{\mathfrak{L}\mathfrak{L}} \mathfrak{L}_{M}^{-1} \end{bmatrix}, \qquad (7.53)$$

and

$$\mathbf{B} = \begin{bmatrix} \mathbb{K}_{CC} \mathbb{C}^{-1} \mathbf{F}_{CS} \mathbf{C}_{S} \mathbf{F}_{VS}^{T} - \mathbb{K}_{CV} \\ \mathbb{K}_{\mathfrak{L}C} \mathbb{C}^{-1} \mathbf{F}_{CS} \mathbf{C}_{S} \mathbf{F}_{VS}^{T} - \mathbb{K}_{V\mathfrak{L}} \end{bmatrix} .$$
(7.54)

7.5.7 Transfer Function Model

An FRA test generates the frequency response of the output voltage relative to the injected input voltage. This is an empirical transfer function estimate (ETFE) between the two test terminals of the FRA test system model. With reference to the normal tree models of Figures 7.4, 7.5 and 7.6, the FRA input voltage is given by v_{IN} and the FRA output is the voltage across the 50 Ω termination resistor R_T . This voltage can be considered in terms of branch capacitor voltage drops. For example, considering the High Voltage Winding End to End Open Circuit FRA test of Figure 7.4, the output voltage is equivalent to the combined voltage drops across the branch capacitors C_{gy1} and C_{Yy1} . The transfer function of the model can then be determined from the resulting input and output voltage relationships.

The capacitor branch voltages can be determined from the state space equation of (7.52) in the following manner. First take the Laplace transform of the state space equation, i.e.

$$\mathbf{s} \begin{bmatrix} \boldsymbol{q}(s) \\ \boldsymbol{\phi}(s) \end{bmatrix} = \mathbf{A} \begin{bmatrix} \boldsymbol{q}(s) \\ \boldsymbol{\phi}(s) \end{bmatrix} + \mathbf{B} \mathbf{v}_{V}(s)$$
$$\mathbf{s} \begin{bmatrix} \boldsymbol{q}(s) \\ \boldsymbol{\phi}(s) \end{bmatrix} - \mathbf{A} \begin{bmatrix} \boldsymbol{q}(s) \\ \boldsymbol{\phi}(s) \end{bmatrix} = \mathbf{B} \mathbf{v}_{V}(s)$$
$$\begin{bmatrix} \boldsymbol{q}(s) \\ \boldsymbol{\phi}(s) \end{bmatrix} = (\mathbf{s} \mathbf{I} - \mathbf{A})^{-1} \mathbf{B} \mathbf{v}_{V}(s) .$$
(7.55)

Since $\mathbf{q}(t)$ is defined as the "net charge per capacitance tree branch cutset" and each of the FRA test models has a common capacitance branch allocation, $\mathbf{q}(t)$ has a matrix dimension relative to the number of capacitance tree branches. With reference to Figures 7.4, 7.5 and 7.6, each section of the proposed *n* section lumped parameter model has branch capacitors C_{gx} , C_{gy} , C_{gz} , C_{Xx} , C_{Yy} and C_{Zz} . As a result $\mathbf{q}(t)$ has a dimension of $6n \times 1$. This dimension is equivalent for $\mathbf{q}(s)$ which can therefore be expressed in terms of a matrix,

$$\boldsymbol{q}(s) = \begin{bmatrix} q(s)_1 & \cdots & q(s)_{6n} \end{bmatrix}^T .$$
(7.56)

Similarly, $\phi(t)$ is defined as the "net flux per inductance link fundamental loop". Noting

that each section of the proposed *n* section lumped parameter model has inductive element links \mathfrak{L}_X , \mathfrak{L}_Y , \mathfrak{L}_Z , \mathfrak{L}_x , \mathfrak{L}_y , and \mathfrak{L}_z , $\phi(t)$ has a matrix dimension of $6n \times 1$. $\phi(s)$ can also be expressed as a matrix,

$$\boldsymbol{\phi}(s) = \begin{bmatrix} \phi(s)_1 & \cdots & \phi(s)_{6n} \end{bmatrix}^T .$$
(7.57)

With reference to each of the system models of Figures 7.4, 7.5 and 7.6, it can be observed that the branch voltage vector for voltage sources, $\mathbf{v}_V(s)$, is the FRA injection voltage $v_{IN}(s)$, i.e.

$$\mathbf{v}_V(s) = v_{IN}(s) \quad . \tag{7.58}$$

Now from (7.55), (7.56) and (7.57), a $12n \times 1$ dimensional matrix **P** can be defined such that,

$$\mathbf{P} = (\mathbf{sI} - \mathbf{A})^{-1} \mathbf{B} = \begin{bmatrix} p_1 & \cdots & p_{12n} \end{bmatrix}^T .$$
(7.59)

Equation (7.59) can be written in terms relative to q(s) and $\phi(s)$,

$$\mathbf{P} = \left[\mathbf{P}_q : \mathbf{P}_\phi\right]^T \quad , \tag{7.60}$$

where

$$\mathbf{P}_q = \begin{bmatrix} p_1 & \cdots & p_{6n} \end{bmatrix} , \qquad (7.61)$$

and

$$\mathbf{P}_{\phi} = \begin{bmatrix} p_{(6n+1)} & \cdots & p_{12n} \end{bmatrix} . \tag{7.62}$$

Rewriting q(s) using (7.55), (7.58), (7.59) and (7.61), gives

$$\boldsymbol{q}(s) = \mathbf{P}_q^T \boldsymbol{v}_{IN}(s) \quad , \tag{7.63}$$

Rewriting $\phi(s)$ using (7.55), (7.58), (7.59) and (7.62), gives

$$\boldsymbol{\phi}(s) = \mathbf{P}_{\phi}^T \boldsymbol{v}_{IN}(s) \quad . \tag{7.64}$$

Equating the Laplace transform of (7.46) with (7.63) and substituting $v_{IN}(s)$ for $\mathbf{v}_V(s)$,

$$\mathbf{P}_{q}^{T} v_{IN}(s) = \mathbb{C} \mathbf{v}_{C}(s) + \mathbf{F}_{CS} \mathbf{C}_{S} \mathbf{F}_{VS}^{T} v_{IN}(s)$$
$$\mathbb{C} \mathbf{v}_{C}(s) = \mathbf{P}_{q}^{T} v_{IN}(s) - \mathbf{F}_{CS} \mathbf{C}_{S} \mathbf{F}_{VS}^{T} v_{IN}(s)$$
$$\mathbf{v}_{C}(s) = \mathbb{C}^{-1} \left[\mathbf{P}_{q}^{T} - \mathbf{F}_{CS} \mathbf{C}_{S} \mathbf{F}_{VS}^{T} \right] v_{IN}(s) .$$
(7.65)

Equation (7.65) facilitates the determination of all of the branch capacitance voltages,

$$\mathbf{v}_{C}(s) = \begin{bmatrix} v_{Cgx2} & \cdots & v_{Cgx(n+1)} & v_{Cgy1} & \cdots & v_{Cgy(n)} & v_{Cgz1} & \cdots & v_{Cgz(n)} \\ v_{CXx1} & \cdots & v_{CXx(n)} & v_{CYy1} & \cdots & v_{CYy(n)} & v_{CZz1} & \cdots & v_{CZz(n)} \end{bmatrix}_{6n \times 1}^{T} (7.66)$$

where $\mathbf{v}_C(s)$ is the branch capacitor voltage matrix for the models given in Figures 7.4, 7.5 and 7.6. To determine $v_{OUT}(s)$ we define a matrix \mathbf{W} of dimension $1 \times 6n$ for the summation of the appropriate branch capacitor voltages such that,

$$v_{OUT}(s) = \mathbf{W}\mathbf{v}_{\mathbf{C}}(s) \quad . \tag{7.67}$$

Substituting (7.65) into (7.67),

$$v_{OUT}(s) = \mathbf{W} \mathbb{C}^{-1} \left[\mathbf{P}_{\mathbf{q}}^{\mathbf{T}} - \mathbf{F}_{\mathbf{CS}} \mathbf{C}_{\mathbf{S}} \mathbf{F}_{\mathbf{VS}}^{\mathbf{T}} \right] v_{IN}(s)$$

$$\therefore \quad G(s) = \frac{v_{OUT}(s)}{v_{IN}(s)} = \mathbf{W} \mathbb{C}^{-1} \left[\mathbf{P}_{\mathbf{q}}^{\mathbf{T}} - \mathbf{F}_{\mathbf{CS}} \mathbf{C}_{\mathbf{S}} \mathbf{F}_{\mathbf{VS}}^{\mathbf{T}} \right] .$$
(7.68)

A transfer function for the FRA system models of Figures 7.4, 7.5 and 7.6 has now been defined in (7.68). By applying a parameter estimation algorithm to FRA data, estimates of the physical parameters of the model can now be obtained.

7.6 Example:

High Voltage Winding End to End Open Circuit FRA test for a Dyn1 vector group between phases A and B.

For an example, this section applies the mathematical model developed in the previous section to a High Voltage Winding End to End Open Circuit FRA test between terminals A and B of a Dyn1 connected power transformer. The normal tree model of Figure 7.4 is applicable, and with reference to Table 7.1, the physical to generic phase substitutions are B to X, A to Y and C to Z. However, physical phase references are only made when the relationship is phase dependent.

Note that the procedure applied in this section can be similarly applied to all of the other FRA test and vector group configurations that have been considered thus far.

7.6.1 Capacitance Matrices

As discussed in Chapter 6, due to the geometric layout of a transformer, capacitance is phase dependent. For example, the interphase high voltage winding capacitance is non-zero between adjacent windings, i.e. capacitors C_{BA} and C_{CB} which represent the capacitance between the centre limb winding (phase B) and the windings on the outer limbs (Phases A and C).

7.6. EXAMPLE:

The capacitor branch matrix, \mathbf{C}_C , was first introduced in equation (7.16). \mathbf{C}_C is a positive definite diagonal matrix that incorporates all of the branch capacitors associated with the FRA model 4 ,

$$\mathbf{C}_{C} = \operatorname{diag} \begin{bmatrix} C_{gx2} & \cdots & C_{gx(n+1)} & C_{gy1} & \cdots & C_{gy(n)} & C_{gz1} & \cdots & C_{gz(n)} \\ C_{Xx1} & \cdots & C_{Xx(n)} & C_{Yy1} & \cdots & C_{Yy(n)} & C_{Zz1} & \cdots & C_{Zz(n)} \end{bmatrix}_{6n \times 6n}$$

$$= \operatorname{diag} \begin{bmatrix} C_{gb2} & \cdots & C_{gb(n+1)} & C_{ga1} & \cdots & C_{ga(n)} & C_{gc1} & \cdots & C_{gc(n)} \\ C_{Bb1} & \cdots & C_{Bb(n)} & C_{Aa1} & \cdots & C_{Aa(n)} & C_{Cc1} & \cdots & C_{Cc(n)} \end{bmatrix}_{6n \times 6n} .$$
(7.69)

Similarly, the elastance link parameter matrix (in terms of capacitance), \mathbf{C}_{S} , is given by 5 ,

$$\begin{split} \mathbf{C}_{S} &= \operatorname{diag} \Big[C_{gx1} \ C_{Xx(n+1)} \ C_{Yy(n+1)} \ C_{Zz(n+1)} \ C_{gy(n+1)} \ C_{gz(n+1)} \\ & C_{SX1} \ \cdots \ C_{SX(n)} \ C_{SY1} \ \cdots \ C_{SY(n)} \ C_{SZ1} \ \cdots \ C_{SZ(n)} \\ & C_{Sx1} \ \cdots \ C_{Sx(n)} \ C_{Sy1} \ \cdots \ C_{Sy(n)} \ C_{Sz1} \ \cdots \ C_{Sz(n)} \\ & C_{XY1} \ \cdots \ C_{XY(n+1)} \ C_{YZ1} \ \cdots \ C_{YZ(n+1)} \ C_{ZX1} \ \cdots \ C_{ZX(n+1)} \\ & C_{gX1} \ \cdots \ C_{gX(n+1)} \ C_{gY1} \ \cdots \ C_{gY(n+1)} \ C_{gZ(n+1)} \\ & \Big]_{(12n+12)\times(12n+12)} \end{split}$$

$$= \operatorname{diag} \begin{bmatrix} C_{gb1} \ C_{Bb(n+1)} \ C_{Aa(n+1)} \ C_{Cc(n+1)} \ C_{ga(n+1)} \ C_{gc(n+1)} \\ C_{SB1} \ \cdots \ C_{SB(n)} \ C_{SA1} \ \cdots \ C_{SA(n)} \ C_{SC1} \ \cdots \ C_{SC(n)} \\ C_{Sb1} \ \cdots \ C_{Sb(n)} \ C_{Sa1} \ \cdots \ C_{Sa(n)} \ C_{Sc1} \ \cdots \ C_{Sc(n)} \\ C_{BA1} \ \cdots \ C_{BA(n+1)} \ C_{AC1} \ \cdots \ C_{AC(n+1)} \ C_{CB1} \ \cdots \ C_{CB(n+1)} \\ C_{gB1} \ \cdots \ C_{gB(n+1)} \ C_{gA1} \ \cdots \ C_{gA(n+1)} \ C_{gC1} \ \cdots \ C_{gC(n+1)} \\ \end{bmatrix}_{(12n+12)\times(12n+12)} .$$

$$(7.70)$$

7.6.2**Inductance Matrices**

In equation (7.24), \mathfrak{L}_M was defined as the tree link inductance matrix. From Chapter 6, it was shown that the transformer inductance consists of both ferromagnetic and leakage

⁴Each section of the *n* section lumped parameter model has the branch capacitors C_{gx} , C_{gy} , C_{gz} , C_{Xx} ,

 C_{Yy} and C_{Zz} . As a result the capacitor branch parameter model has the branch capacitors C_{gx} , C_{gy} , C_{gz} , C_{Xx} , 5 The *n* section lumped parameter model has the link capacitors C_{gx1} , $C_{Xx(n+1)}$, $C_{Yy(n+1)}$, $C_{Zz(n+1)}$, $C_{gy(n+1)}$, $C_{gz(n+1)}$, as well as *n* sections of C_{SX} , C_{SY} , C_{SZ} , C_{Sx} , C_{Sy} , C_{Sz} and (n+1) sections of C_{XY} , C_{YZ} , C_{ZX} , C_{gX} , C_{gY} , C_{gZ} . This results in an elastance link parameter matrix with a dimension of $(12n+12) \times (12n+12)$ $(12n + 12) \times (12n + 12).$



Figure 7.7: 3 phase transformer mutual flux linkages. Flux linkage with respect to current injection into (a) phase A, (b) phase B, (c) phase C.

inductance components. Hence \mathfrak{L}_M can be written as,

$$\mathfrak{L}_M = \mathbf{L}_F + \mathbf{L}_L \quad , \tag{7.71}$$

where \mathbf{L}_F is a matrix for the ferromagnetic inductance and \mathbf{L}_L is a matrix for the leakage inductance. Both \mathbf{L}_F and \mathbf{L}_L are comprised of self and mutual inductance elements. Noting that mutual inductance relates the flux linkage on one winding due to the excitation of another, it is then important to consider the flux direction within the various limbs of a core type transformer.

With reference to Figure 7.7, taking the reference current and winding directions to be the same for all windings, it can be observed that flux generated by a winding on one phase will flow in the opposite direction through the windings of the other two phases. Conversely, windings on the same limb will observe the same flux direction. On this basis, the mutual inductance between winding sections on different phases is considered negative and the mutual inductance between winding sections on the same phase is considered positive. Taking into account the self and mutual inductance relationships and their associated reference directions, the ferromagnetic matrix, \mathbf{L}_F , is ⁶,

$$\mathbf{L}_{F} = \begin{bmatrix} +\mathbf{L}_{XiXj} & -\mathbf{L}_{XiYj} & -\mathbf{L}_{XiZj} & +\mathbf{L}_{Xixj} & -\mathbf{L}_{Xiyj} & -\mathbf{L}_{Xizj} \\ -\mathbf{L}_{YiXj} & +\mathbf{L}_{YiYj} & -\mathbf{L}_{YiZj} & -\mathbf{L}_{Yixj} & +\mathbf{L}_{Yiyj} & -\mathbf{L}_{Yizj} \\ -\mathbf{L}_{ZiXj} & -\mathbf{L}_{ZiYj} & +\mathbf{L}_{ZiZj} & -\mathbf{L}_{Zixj} & -\mathbf{L}_{Ziyj} & +\mathbf{L}_{Zizj} \\ +\mathbf{L}_{xiXj} & -\mathbf{L}_{xiYj} & -\mathbf{L}_{xiZj} & +\mathbf{L}_{xixj} & -\mathbf{L}_{xiyj} & -\mathbf{L}_{xizj} \\ -\mathbf{L}_{yiXj} & +\mathbf{L}_{yiYj} & -\mathbf{L}_{yiZj} & -\mathbf{L}_{yixj} & +\mathbf{L}_{yiyj} & -\mathbf{L}_{yizj} \\ -\mathbf{L}_{ziXj} & -\mathbf{L}_{ziYj} & +\mathbf{L}_{ziZj} & -\mathbf{L}_{zixj} & -\mathbf{L}_{ziyj} & +\mathbf{L}_{zizj} \end{bmatrix}_{6n\times6n} \\ = \begin{bmatrix} +\mathbf{L}_{BiBj} & -\mathbf{L}_{BiAj} & -\mathbf{L}_{BiCj} & +\mathbf{L}_{Bibj} & -\mathbf{L}_{Biaj} & -\mathbf{L}_{Bicj} \\ -\mathbf{L}_{AiBj} & +\mathbf{L}_{AiAj} & -\mathbf{L}_{AiCj} & -\mathbf{L}_{Aibj} & +\mathbf{L}_{Aiaj} & -\mathbf{L}_{Aicj} \\ -\mathbf{L}_{CiBj} & -\mathbf{L}_{CiAj} & +\mathbf{L}_{CiCj} & -\mathbf{L}_{Cibj} & -\mathbf{L}_{Ciaj} & +\mathbf{L}_{Cicj} \\ +\mathbf{L}_{biBj} & -\mathbf{L}_{biAj} & -\mathbf{L}_{biCj} & +\mathbf{L}_{bibj} & -\mathbf{L}_{biaj} & -\mathbf{L}_{bicj} \\ -\mathbf{L}_{aiBj} & +\mathbf{L}_{aiAj} & -\mathbf{L}_{aiCj} & -\mathbf{L}_{aibj} & +\mathbf{L}_{aiaj} & -\mathbf{L}_{aicj} \\ -\mathbf{L}_{ciBj} & -\mathbf{L}_{ciAj} & +\mathbf{L}_{ciCj} & -\mathbf{L}_{cibj} & -\mathbf{L}_{biaj} & -\mathbf{L}_{bicj} \\ -\mathbf{L}_{ciBj} & -\mathbf{L}_{ciAj} & +\mathbf{L}_{ciCj} & -\mathbf{L}_{cibj} & -\mathbf{L}_{biaj} & -\mathbf{L}_{bicj} \\ -\mathbf{L}_{ciBj} & -\mathbf{L}_{ciAj} & +\mathbf{L}_{ciCj} & -\mathbf{L}_{cibj} & -\mathbf{L}_{ciaj} & +\mathbf{L}_{cicj} \end{bmatrix}_{6n\times6n} \end{cases}$$

for $i, j \in [1..n]$. The resulting submatrix elements of \mathbf{L}_F are $n \times n$ square matrices based on the transformer inductance matrix of Table 6.1. Since it is assumed that the ferromagnetic coupling across a winding section is uniform, with reference to Table 6.1, \mathbf{L}_F becomes,

$$\mathbf{L}_{F} = \begin{bmatrix} +\frac{\bar{L}}{\Gamma} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma} [\mathbf{1}] & +\frac{\bar{L}}{\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] \\ -\frac{\bar{L}}{2\Gamma} [\mathbf{1}] & +\bar{L} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & +\frac{\bar{L}}{\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda\bar{a}} [\mathbf{1}] \\ -\frac{\bar{L}}{2\Gamma} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda} [\mathbf{1}] & +\bar{L} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda\bar{a}} [\mathbf{1}] & +\frac{\bar{L}}{\bar{a}} [\mathbf{1}] \\ +\frac{\bar{L}}{\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}^2} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}^2} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}^2} [\mathbf{1}] \\ -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & +\frac{\bar{L}}{\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}^2} [\mathbf{1}] & +\frac{\bar{L}}{\bar{a}^2} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda\bar{a}^2} [\mathbf{1}] \\ -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}^2} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda\bar{a}^2} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}^2} [\mathbf{1}] \\ -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\Lambda\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}^2} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}^2} [\mathbf{1}] \\ -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{2\Gamma\bar{a}^2} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}^2} [\mathbf{1}] \\ -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}^2} [\mathbf{1}] \\ -\frac{\bar{L}}{2\Gamma\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] \\ -\frac{\bar{L}}{2\bar{A}\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] & -\frac{\bar{L}}{\bar{A}\bar{a}} [\mathbf{1}] \\ -\frac{\bar$$

⁶Each section of the *n* section lumped parameter model has the tree link inductance parameters \mathfrak{L}_X , \mathfrak{L}_Y , \mathfrak{L}_Z , \mathfrak{L}_x , \mathfrak{L}_y , and \mathfrak{L}_z . Since each tree link inductance is comprised of both a ferromagnetic and a leakage inductance contribution, matrices \mathbf{L}_F and \mathbf{L}_L will have a dimension of $6n \times 6n$.

where [1] is an $n \times n$ matrix where all entries are 1. \mathbf{L}_F is the ferromagnetic inductance matrix and therefore the leakage inductance of Table 6.1 is not included. The distributed ferromagnetic base inductance \bar{L} , as defined in (6.1), is a frequency dependent term based upon the permeability relationship of (3.22).

From Section 6.3.2, leakage inductance is assumed to have no cross coupling between windings. This results in a leakage inductance matrix \mathbf{L}_L that is diagonal in terms of its submatrix elements,

$$\mathbf{L}_{L} = \begin{bmatrix} \mathbf{L}_{LXij} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_{LYij} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{L}_{LZij} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_{Lxij} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_{Lyij} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_{Lzij} \end{bmatrix}_{6n \times 6n} \\ = \begin{bmatrix} \mathbf{L}_{LBij} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_{LAij} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{L}_{LCij} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_{Lbij} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_{Laij} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_{Lcij} \end{bmatrix}_{6n \times 6n} , \quad (7.74)$$

for $i, j \in [1..n]$. Each of the submatrix elements of \mathbf{L}_L are $n \times n$ in size but unlike \mathbf{L}_F , the diagonal (non-zero) components are not uniform and are governed by the distributed leakage inductance relationship of equation (6.11), which is

$$L_{LXij} \approx \frac{\tau^{|i-j|} L_{LX}}{\left[\sum_{i=1}^n \sum_{j=1}^n \tau^{|i-j|}\right]} \quad \text{for } 0 \leq \tau \leq 1 \text{ and } i, j \in [1..n].$$

As an example, the submatrix \mathbf{L}_{LBij} is given by,

$$\mathbf{L}_{LBij} \approx \frac{1}{\left[\sum_{i=1}^{n} \sum_{j=1}^{n} \tau^{|i-j|}\right]} \times \begin{bmatrix} L_{LB} & \tau^{1}L_{LB} & \tau^{2}L_{LB} & \cdots & \tau^{(n-1)}L_{LB} \\ \tau^{1}L_{LB} & L_{LB} & \tau^{1}L_{LB} & \cdots & \tau^{(n-2)}L_{LB} \\ \tau^{2}L_{LB} & \tau^{1}L_{LB} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \tau^{1}L_{LB} \\ \tau^{(n-1)}L_{LB} & \tau^{(n-2)}L_{LB} & \cdots & \tau^{1}L_{LB} & L_{LB} \end{bmatrix}_{n \times n}, (7.75)$$

where $0 \le \tau \le 1$.

7.6.3 Conductance and Resistance Matrices

For mathematical convenience the series resistance element of the winding for the generic phase model, as defined in Section 6.4, was converted to its conductance form in (7.2). With reference to Figure 7.4, all of the conductance elements within the model's normal tree are "branches". As such, the conductive element branch parameter matrix, \mathfrak{G}_M , that was introduced in equation (7.26), is given by⁷,

$$\mathbf{\mathfrak{G}}_{M} = \operatorname{diag} \begin{bmatrix} \mathfrak{G}_{X1} & \cdots & \mathfrak{G}_{X(n)} & \mathfrak{G}_{Y1} & \cdots & \mathfrak{G}_{Y(n)} & \mathfrak{G}_{Z1} & \cdots & \mathfrak{G}_{Z(n)} \\ \mathbf{\mathfrak{G}}_{x1} & \cdots & \mathbf{\mathfrak{G}}_{x(n)} & \mathbf{\mathfrak{G}}_{y1} & \cdots & \mathbf{\mathfrak{G}}_{y(n)} & \mathbf{\mathfrak{G}}_{z1} & \cdots & \mathbf{\mathfrak{G}}_{z(n)} \end{bmatrix}_{6n \times 6n} \\ = \operatorname{diag} \begin{bmatrix} \mathfrak{G}_{B1} & \cdots & \mathfrak{G}_{B(n)} & \mathfrak{G}_{A1} & \cdots & \mathfrak{G}_{A(n)} & \mathfrak{G}_{C1} & \cdots & \mathfrak{G}_{C(n)} \\ \mathbf{\mathfrak{G}}_{b1} & \cdots & \mathbf{\mathfrak{G}}_{b(n)} & \mathfrak{\mathfrak{G}}_{a1} & \cdots & \mathfrak{G}_{a(n)} & \mathfrak{G}_{c1} & \cdots & \mathfrak{G}_{c(n)} \end{bmatrix}_{6n \times 6n} .$$
(7.76)

Once again for mathematical convenience, the capacitor dielectric loss conductance of Section 6.5.6 was converted to resistance in (7.1). These dielectric loss resistances, plus the addition of the FRA termination resistor R_T , are "links" in the normal tree of Figure

⁷Each section of the *n* section lumped parameter model has the series conductance branch parameters $\mathfrak{G}_X, \mathfrak{G}_Y, \mathfrak{G}_Z, \mathfrak{G}_x, \mathfrak{G}_y$ and \mathfrak{G}_z . As a result the conductive element branch parameter matrix will have a dimension of $6n \times 6n$.

7.6. EXAMPLE:

7.4. The tree link resistance matrix \mathbf{R}_R is given by ⁸,

$$\begin{split} \mathbf{R}_{R} &= \text{diag} \bigg[R_{SX1} & \cdots & R_{SX(n)} \ R_{SY1} & \cdots & R_{SY(n)} \ R_{SZ1} & \cdots & R_{SZ(n)} \\ & R_{Sx1} & \cdots & R_{Sx(n)} \ R_{Sy1} & \cdots & R_{Sy(n)} \ R_{Sz1} & \cdots & R_{Sz(n)} \\ & R_{gX1} & \cdots & R_{gX(n+1)} \ R_{gY1} & \cdots & R_{gY(n+1)} \ R_{gZ1} & \cdots & R_{gZ(n+1)} \\ & R_{gx1} & \cdots & R_{gx(n+1)} \ R_{gy1} & \cdots & R_{gy(n+1)} \ R_{gz1} & \cdots & R_{gz(n+1)} \\ & R_{Xx1} & \cdots & R_{Xx(n+1)} \ R_{Yy1} & \cdots & R_{Yy(n+1)} \ R_{Zz1} & \cdots & R_{Zz(n+1)} \ R_{T} \\ & \bigg] (15n+10) \times (15n+10) \end{split}$$

$$= \operatorname{diag} \left[R_{SB1} \cdots R_{SB(n)} \ R_{SA1} \cdots R_{SA(n)} \ R_{SC1} \cdots R_{SC(n)} \right] \\ R_{Sb1} \cdots R_{Sb(n)} \ R_{Sa1} \cdots R_{Sa(n)} \ R_{Sc1} \cdots R_{Sc(n)} \\ R_{gB1} \cdots R_{gB(n+1)} \ R_{gA1} \cdots R_{gA(n+1)} \ R_{gC1} \cdots R_{gC(n+1)} \\ R_{gb1} \cdots R_{gb(n+1)} \ R_{ga1} \cdots R_{ga(n+1)} \ R_{gc1} \cdots R_{gc(n+1)} \\ R_{Bb1} \cdots R_{Bb(n+1)} \ R_{Aa1} \cdots R_{Aa(n+1)} \ R_{Cc1} \cdots R_{Cc(n+1)} \ R_{T} \\ \left] (15n+10) \times (15n+10) \ . \end{array}$$

$$(7.77)$$

As a minor simplification, the resistance elements for the interwinding dielectric loss have been neglected at this stage.

7.6.4 F Submatrices

In Section 7.5.1, the tree branch to link coupling matrix \mathbf{F} was given in (7.7). In (7.7), the matrix rows correspond to individual tree branches and the matrix columns correspond to individual tree links. The value of a matrix cell is determined by the application of a fundamental cutset to each branch (row). All of the unaffected links will have a branch-link cell value of 0. Of the "cut" links, if the link and branch have the same graph polarity relative to the cutset, the branch-link cell will have a value of +1. For the opposite polarity, the cell value is -1.

The matrix rows follow the branch allocation order discussed in Section 7.4 with respect to the model in Figure 7.4. In this example the test voltage source v_{IN} is followed by the branch capacitors (diagonal elements of \mathbf{C}_C in (7.69)), followed by the branch conductances (diagonal elements of $\mathbf{\mathfrak{G}}_M$ in (7.76)). Table 7.4 shows the alignment between the normal tree branches of Figure 7.4, and the corresponding rows of matrix⁹ **F**.

Similarly, each column of \mathbf{F} in (7.7) represents a link within the normal tree of Figure

⁸The *n* section lumped parameter model has the tree link resistor R_T , as well as *n* sections of R_{SX} , R_{SY} , R_{SZ} , R_{Sx} , R_{Sy} , R_{Sz} , and (n+1) sections of R_{gX} , R_{gY} , R_{gZ} , R_{gx} , R_{gy} , R_{gz} , R_{Xx} , R_{Yy} , R_{Zz} . This results in a tree link resistance matrix with a dimension of $(15n + 10) \times (15n + 10)$.

⁹Generic phase references are used throughout this section since the branch to link coupling matrix is the same for all phase orientations of the High Voltage Winding End to End Open Circuit FRA test.

Row	Branch	Row	Branch	Row	Branch
1	v_{IN}	4n+2	C_{Yy1}	:	:
2	C_{gx2}	:	:	9n + 1	$\mathfrak{G}_{Z(n)}$
÷	÷	5n+1	$C_{Yy(n)}$	9n+2	\mathfrak{G}_{x1}
n+1	$C_{gx(n+1)}$	5n+2	C_{Zz1}	÷	:
n+2	C_{gy1}	:	÷	10n + 1	$\mathfrak{G}_{x(n)}$
:	:	6n + 1	$C_{Zz(n)}$	10n + 2	\mathfrak{G}_{y1}
2n + 1	$C_{gy(n)}$	6n+2	\mathfrak{G}_{X1}	÷	:
2n+2	C_{gz1}	:	÷	11n + 1	$\mathfrak{G}_{y(n)}$
:	:	7n+1	$\mathfrak{G}_{X(n)}$	11n + 2	\mathfrak{G}_{z1}
3n + 1	$C_{gz(n)}$	7n+2	\mathfrak{G}_{Y1}	÷	:
3n+2	C_{Xx1}	:	÷	12n + 1	$\mathfrak{G}_{z(n)}$
:	:	8n + 1	$\mathfrak{G}_{Y(n)}$	_	-
4n + 1	$C_{Xx(n)}$	8n+2	\mathfrak{G}_{Z1}	-	-

Table 7.4: Alignment between the normal tree branches of Figure 7.4 and the rows of matrix ${\bf F}$

7.4. The allocation order for the columns of **F** in this example are given in Tables 7.5, 7.6 and 7.7.

With reference to Table 7.4 it is observed that matrix \mathbf{F} has (12n + 1) rows. With reference to Tables 7.5, 7.6 and 7.7 it is observed that matrix \mathbf{F} has (33n + 22) columns. Hence the matrix dimension of \mathbf{F} for this transformer model is $(12n + 1) \times (33n + 22)$. In practice \mathbf{F} is generated in software via algorithmic sequences for each branch link combination. Due to its relatively large size, only the derivation of row one is demonstrated. The remaining rows can be derived in a similar fashion for each FRA test.

7.6.5 Cutset Example: Branch v_{IN} (Row 1)¹⁰

To examine the application of a cutset to branch v_{IN} , which is the first row of **F**, a zoomed in view of the relevant portions of Figure 7.4 is shown in Figure 7.8. The first step is to consider the capacitive links to determine \mathbf{F}_{VS} of (7.7). The affected links (columns) and their respective polarity relative to the cutset are as follows,

$$+ C_{gx1}, + C_{Sx1}, + C_{SX1}, + C_{XY1}, + C_{gX1}, - C_{ZX1}, + C_{ZX(n+1)}, + C_{gZ(n+1)}, - C_{YZ(n+1)}, - C_{SZ(n)}, + C_{Zz(n+1)}$$
(7.78)

¹⁰Note that for consistency, the column index that is used in this section is relative to the matrix \mathbf{F} column position as specified in the respective Tables 7.5, 7.6 and 7.7.

Column	Link	Column	Link	Column	Link
1	C_{gx1}	3n+6	$C_{SZ(n)}$:	:
2	$C_{Xx(n+1)}$	3n + 7	C_{Sx1}	8n + 8	$C_{YZ(n+1)}$
3	$C_{Yy(n+1)}$	÷	÷	8n + 9	C_{ZX1}
4	$C_{Zz(n+1)}$	4n + 6	$C_{Sx(n)}$	÷	:
5	$C_{gy(n+1)}$	4n + 7	C_{Sy1}	9n + 9	$C_{ZX(n+1)}$
6	$C_{gz(n+1)}$	÷	:	9n + 10	C_{gX1}
7	C_{SX1}	5n + 6	$C_{Sy(n)}$:	:
:	÷	5n + 7	C_{Sz1}	10n + 10	$C_{gX(n+1)}$
n+6	$C_{SX(n)}$	÷	:	10n + 11	C_{gY1}
n+7	C_{SY1}	6n + 6	$C_{Sz(n)}$	÷	
:	:	6n + 7	C_{XY1}	11n + 11	$C_{gY(n+1)}$
2n + 6	$C_{SY(n)}$	÷	÷	11n + 12	C_{gZ1}
2n+7	C_{SZ1}	7n + 7	$C_{XY(n+1)}$		
:		7n + 8	C_{YZ1}	12n + 12	$C_{gZ(n+1)}$

Table 7.5: Alignment between the normal tree capacitance links and the corresponding columns of ${\bf F}$



Figure 7.8: Zoomed in view of Figure 7.4 with a cutset applied to branch v_{IN} .

Column	Link	Column	Link	Column	Link
12n + 13	R_{SX1}	:	:	23n + 17	$R_{gy(n+1)}$
:	:	18n + 12	$R_{Sz(n)}$	23n + 18	R_{gz1}
13n + 12	$R_{SX(n)}$	18n + 13	R_{gX1}	÷	÷
13n + 13	R_{SY1}	÷	÷	24n + 18	$R_{gz(n+1)}$
÷	÷	19n + 13	$R_{gX(n+1)}$	24n + 19	R_{Xx1}
14n + 12	$R_{SY(n)}$	19n + 14	R_{gY1}	÷	÷
14n + 13	R_{SZ1}	÷	÷	25n + 19	$R_{Xx(n+1)}$
:	÷	20n + 14	$R_{gY(n+1)}$	25n + 20	R_{Yy1}
15n + 12	$R_{SZ(n)}$	20n + 15	R_{gZ1}	÷	÷
15n + 13	R_{Sx1}	÷	÷	26n + 20	$R_{Yy(n+1)}$
:	÷	21n + 15	$R_{gZ(n+1)}$	26n + 21	R_{Zz}
16n + 12	$R_{Sx(n)}$	21n + 16	R_{gx1}	÷	÷
16n + 13	R_{Sy1}	÷	÷	27n + 21	$R_{Zz(n+1)}$
÷	:	22n + 16	$R_{gx(n+1)}$	27n + 22	R_T
17n + 12	$R_{Sy(n)}$	22n + 17	R_{gy1}	-	_
17n + 13	R_{Sz1}	:	:	-	-

Table 7.6: Alignment between the normal tree resistance links and the corresponding columns of ${\bf F}$

Column	Link	Column	Link	Column	Link
27n + 23	\mathfrak{L}_{X1}	29n + 23	\mathfrak{L}_{Z1}	31n + 23	\mathfrak{L}_{y1}
:	÷	:	÷	÷	•
28n + 22	$\mathfrak{L}_{X(n)}$	30n + 22	$\mathfrak{L}_{Z(n)}$	32n + 22	$\mathfrak{L}_{y(n)}$
28n + 23	\mathfrak{L}_{Y1}	30n + 23	\mathfrak{L}_{x1}	32n + 23	\mathfrak{L}_{z1}
:	÷	÷	÷	÷	:
29n + 22	$\mathfrak{L}_{Y(n)}$	31n + 22	$\mathfrak{L}_{x(n)}$	33n + 22	$\mathfrak{L}_{z(n)}$

Table 7.7: Alignment between the normal tree inductive element links and the corresponding columns of ${\bf F}$

From (7.78) and Table 7.5, for the **F** matrix column order of the capacitor links, the voltage branch to capacitance link submatrix \mathbf{F}_{VS} is given by ¹¹,

$$\mathbf{F}_{VS} = \begin{bmatrix} 1_1 & 0_2 & 0_3 & 1_4 & 0_5 & 0_6 & 1_7 & 0_8 & \cdots & 0_{(3n+5)} & -1_{(3n+6)} & 1_{(3n+7)} \\ 0_{(3n+8)} & \cdots & 0_{(6n+6)} & 1_{(6n+7)} & 0_{(6n+8)} & \cdots & 0_{(8n+7)} & -1_{(8n+8)} \\ & -1_{(8n+9)} & 0_{(8n+10)} & \cdots & 0_{(9n+8)} & 1_{(9n+9)} & 1_{(9n+10)} & 0_{(9n+11)} \cdots \\ & 0_{(12n+11)} & 1_{(12n+12)} \end{bmatrix}_{1 \times (12n+12)} .$$

$$(7.79)$$

Similarly, for the same cutset the resistive links and their respective polarity relative to the cutset are as follows,

$$+R_{gx1}, +R_{Sx1}, +R_{SX1}, +R_{gX1}, +R_{gZ(n+1)}, -R_{SZ(n)}, +R_{ZZ(n+1)} .$$
(7.80)

With reference to Table 7.6, this translates into a voltage branch to resistive link submatrix \mathbf{F}_{VR} of,

$$\mathbf{F}_{VR} = \begin{bmatrix} 1_{(12n+13)} & 0_{(12n+14)} & \cdots & 0_{(15n+11)} & -1_{(15n+12)} & 1_{(15n+13)} & 0_{(15n+14)} \\ & \cdots & 0_{(18n+12)} & 1_{(18n+13)} & 0_{(18n+14)} & \cdots & 0_{(21n+14)} & 1_{(21n+15)} & 1_{(21n+16)} \\ & & 0_{(21n+17)} & \cdots & 0_{(27n+20)} & 1_{(27n+21)} & 0_{(27n+22)} \end{bmatrix}_{1 \times (15n+10)} .$$
(7.81)

The inductive links and their respective polarity relative to the cutset of v_S are,

$$+ \mathfrak{L}_{x1}, + \mathfrak{L}_{X1}, - \mathfrak{L}_{Z(n)} .$$

$$(7.82)$$

With reference to Table 7.7, this translates into a voltage branch to inductive element link submatrix $\mathbf{F}_{V\mathcal{L}}$ of,

$$\mathbf{F}_{V\mathfrak{L}} = \begin{bmatrix} 1_{(27n+23)} & 0_{(27n+24)} & \cdots & 0_{(30n+21)} & -1_{(30n+22)} \\ 1_{(30n+23)} & 0_{(30n+24)} & \cdots & 0_{(33n+22)} \end{bmatrix}_{1\times 6n} .$$
(7.83)

Submatrices \mathbf{F}_{VS} (7.79), \mathbf{F}_{VR} (7.81) and $\mathbf{F}_{V\mathfrak{L}}$ (7.83) make up the first row of the tree branch to link coupling matrix \mathbf{F} (7.7) for the High Voltage Winding End to End Open Circuit FRA test model. The remaining entries of \mathbf{F} can be determined in a similar fashion through the application of a cutset to each of the branch elements. In addition, applying this approach to the network trees of Figures 7.5 and 7.6 will facilitate the determination of the coupling matrix \mathbf{F} for the Low Voltage Winding End to End Open Circuit and Capacitive Interwinding FRA test models.

¹¹A matrix position index is used in the following sections to clearly identify each matrix entry's location. For example, the entry $-1_{(3n+6)}$ indicates that for the n section model the row matrix entry at position (3n+6) is -1.

7.6.6 Determining the Output Voltage

The proposed method used to determine $v_{OUT}(s)$, which is the voltage across R_T , is to determine the voltage across the branch capacitors. Mathematically, this approach is defined in (7.67). In this relationship, the matrix **W** is used to sum the appropriate branch capacitors of $\mathbf{v}_C(s)$. For the system model of Figure 7.4, v_{OUT} is the combined voltage across the branch capacitors C_{gy1} and C_{Yy1} . From 7.66,

$$\mathbf{v}_{C}(s) = \begin{bmatrix} v_{Cgx2} & \cdots & v_{Cgx(n+1)} & v_{Cgy1} & \cdots & v_{Cgy(n)} & v_{Cgz1} & \cdots & v_{Cgz(n)} \\ v_{CXx1} & \cdots & v_{CXx(n)} & v_{CYy1} & \cdots & v_{CYy(n)} & v_{CZz1} & \cdots & v_{CZz(n)} \end{bmatrix}_{6n \times 1}^{T} (7.84)$$

Therefore matrix \mathbf{W} can be defined as,

$$\mathbf{W} = \begin{bmatrix} 0_1 & \cdots & 0_n & 1_{(n+1)} & 0_{(n+2)} & \cdots & 0_{4n} & 1_{(4n+1)} & 0_{(4n+2)} & \cdots & 0_{6n} \end{bmatrix}_{1 \times 6n} .$$
(7.85)

The example has shown how to generate the relevant matrix relationships required to obtain the transfer function between terminals A and B of a model based on Figure 7.4. This same approach can be followed in order to generate any of the FRA test transfer functions based on the models given in Figures 7.4, 7.5 and 7.6.

7.7 Conclusion

The aim of this chapter was to develop flexible power transformer models based on a number of different types of FRA tests. To achieve this, a layered modelling approach was adopted. The first layer consisted of three instances of the generic phase model derived in Chapter 6. The second layer was the transformer vector group where both Dyn1 and Dyn11 connections were considered. The third layer was the prescribed FRA test connection. For this layer, three different types of FRA test were considered, i.e. the High Voltage End to End Open Circuit, the Low Voltage End to End Open Circuit and the Capacitive Interwinding tests.

The resulting models were then converted into a mathematical form by producing an equivalent network tree for each type of FRA test. This facilitated the generation of a state space representation which was then converted into a transfer function.

Taking into account the three terminal permutations, the three types of FRA test, and the two different vector groups that are considered, a generic transfer function was developed that can have up to 18 individual variants (9 for Dyn1 and 9 for Dyn11). The procedural steps required to generate each of the individual transfer functions is the same. In an example to demonstrate the procedure, the transfer function of the High Voltage Winding End to End Open Circuit FRA test between terminals A and B of a Dyn1 connected transformer was determined. In this example each of the parameter matrices was defined and the derivation of the branch to link coupling matrix was given.

Chapter 8

Initial Parameter Estimates and Constraints

8.1 Introduction

In this chapter we formulate a methodology to obtain initial estimates and constraints for several transformer parameters. Each of these is based on readily available information such as transformer nameplate details, routine test data, external dimensions, and common transformer manufacturing design rules. The constraints provide the estimation algorithm (as discussed in Chapter 9) with upper and lower bounds to ensure that the parameter solution is physically feasible. The initial estimate is used to seed the parameter estimation algorithm, and in conjunction with the constraints, serves to prevent the estimation algorithm converging to a local minimum. To demonstrate applicability, the initial parameter estimates and constraints in the following sections are applied to a range of power transformers of varying age and manufacture, ranging in size from 200kVA to 18MVA.

This chapter is structured in the following manner. Section 8.2 highlights the importance of having a constrained estimator when fitting the proposed transformer model to a data set. It also examines the difficulties related to obtaining detailed internal specifications of a power transformer. Section 8.3 derives the initial parameter estimate and constraints for the transformer winding conductor cross sectional area. Section 8.4 derives a relationship which determines the winding resistance from a resistance test between terminals for delta and star connection topologies. Section 8.5 derives the initial parameter estimate and constraints for the winding conductor length. Section 8.6 derives the initial parameter estimate and constraints for the mean diameter of the HV winding. Sections 8.7 and 8.8 derive the initial parameter estimate and constraints for the number of turns on the HV and LV windings respectively. Section 8.9 derives the initial parameter estimate and constraints for the dimensions of the transformer core. Section 8.10 derives the initial parameter estimate and constraints for the transformer core area. Concluding remarks are then given in Section 8.11.

8.2 Background

A fundamental goal of this research is to develop a power transformer model which can be used to interpret the results obtained from frequency response analysis. During the process of fitting a model to a data set, it is imperative to ensure that the estimated parameters are physically representative of the transformer under test. An estimator that is not constrained can converge on a parameter set which may satisfy the objective function but is not physically representative of the transformer [30]. It is therefore important to constrain the model parameters. This can be achieved by incorporating as many of the known electrical and mechanical properties of the transformer as possible, into the estimation algorithm. Manufacturer's proprietary restrictions will generally make access to detailed design drawings difficult. In addition, the world wide power transformer population is relatively aged (the average age of a power transformer in the United States is almost 40 years [90]). Obtaining relevant design documentation for equipment built decades ago can be quite a problem. It is well known that failure probability increases significantly in the final quartile of a transformer's life [90], hence it is the older transformers where routine condition monitoring is most critical and the lack of design documentation is most prevalent. In this light, we seek to utilise all of the readily available information in order to determine an appropriate initial estimate and corresponding set of constraints for use in an estimation algorithm in Chapter 9.

8.3 Cross Sectional Area of Winding Conductors

The winding conductors of an oil filled power transformer are typically designed to have an operating current density in the range of 2 to 4 A/mm^2 [53]. Hence for the majority of transformers it is possible to obtain upper and lower bounds on the conductor cross sectional area based on the transformer winding's current rating. On this basis, for a star connected winding, constraints on the conductor cross sectional area can be given by,

$$[A_{CL}, A_{CU}] = \frac{i_{HR}}{[4, 2]} \text{ mm}^2 , \qquad (8.1)$$

where A_{CL} and A_{CU} represent the lower and upper bounds respectively and i_{HR} is the current rating of the winding. For a delta connection, taking into account the winding current relative to the terminal current, constraints on the conductor cross sectional area are given by,

$$[A_{CL}, A_{CU}] = \frac{i_{HR}}{\sqrt{3}[4, 2]} \text{ mm}^2 .$$
(8.2)
A generic form of (8.1) and (8.2) can be formulated by utilising a scaling factor κ where,

$$\kappa = \begin{cases}
1 & \text{Star connected winding} \\
\sqrt{3} & \text{Delta connected winding}.
\end{cases}$$
(8.3)

Therefore, the generic form of (8.1) and (8.2) is,

$$[A_{CL}, A_{CU}] = \frac{i_{HR}}{2\kappa} \left[\frac{1}{2}, 1\right] \text{ mm}^2 .$$
(8.4)

A convenient initial parameter estimate for the conductor cross sectional area is the midpoint of the constraints,

$$\hat{A}_C = \frac{3i_{HR}}{8\kappa} \ \mathrm{mm}^2 \ . \tag{8.5}$$

To demonstrate the veracity of the relationships (8.4) and (8.5), an example follows.

Example : Calculation of the initial parameter estimate and constraints for the conductor cross sectional area of the high and low voltage windings of a Dyn1 1.3MVA 11kV/433V distribution transformer.

The vector group for this distribution transformer is Dyn1 hence the high voltage side is delta connected. The nameplate rating for the HV current is 73A. Substituting the nameplate rating into (8.4), where $\kappa = \sqrt{3}$ for the delta connection, yields the following constraints,

$$[A_{CL}, A_{CU}]_{HV} = \frac{73}{2 \times \sqrt{3}} \left[\frac{1}{2}, 1 \right]$$

= [10.5, 21] mm². (8.6)

From (8.5), the corresponding initial parameter estimate is,

$$[\hat{A}_C]_{HV} = \frac{3 \times 73}{8 \times \sqrt{3}}$$

= 15.8 mm². (8.7)

The actual high voltage winding conductor cross sectional area is 13.6 mm^2 . This value is within the constraints specified in (8.6). The initial parameter estimate of 15.8 mm^2 has an error relative to the actual cross sectional area of 16%. Application of (8.4) and (8.5) to a star connected winding, is suitably demonstrated via the distribution transformer's secondary. The nameplate current rating is 1777 A. The corresponding

Transformer Description	Act.	LB	Est.	UB	Err. (%)
18MVA 33kV/6.6kV Dyn11	72.45	45.45	68.18	90.91	6
15MVA 33kV/11kV Dyn11	62.50	37.88	56.82	75.76	9
5MVA 66kV/11kV Dyn11	11.62	6.31	9.47	12.63	19
1.3MVA 11kV/433V Dyn1	13.60	10.54	15.80	21.07	16
200kVA 11kV/433kV Dyn11	2.80	1.52	2.27	3.03	19

Table 8.1: High voltage winding conductor cross sectional area for several power transformers. Dimensions are in mm^2 .

constraints based on a star connection are,

$$[A_{CL}, A_{CU}]_{LV} = \frac{1777}{2 \times 1} \left[\frac{1}{2}, 1\right] \text{ mm}^2 .$$

= [444, 889] mm². (8.8)

The secondary winding conductor cross sectional area initial parameter estimate is,

$$[\hat{A}_C]_{LV} = \frac{3 \times 1777}{8 \times 1}$$

= 666 mm². (8.9)

The actual low voltage winding conductor cross sectional area is 710 mm^2 which is within the constraints specified in (8.8). The relative error of the initial parameter estimate to the actual cross sectional area is 6%.

The Act. column of Table 8.1 lists the high voltage winding conductor cross sectional areas of five different power transformers. The parameter lower bound, LB, and upper bound, UB, are given for each case. All examples are within the specified limits. The initial parameter estimates are listed in the Est. column. The relative error between the actual cross sectional area and the initial parameter estimate is listed in the Err. column as a percentage. The worst case error is 19%.

This section has defined the initial parameter estimate and constraints for the winding conductor cross sectional area of a transformer. The relationships were applied to power transformers, of various size, and resulted in the constraints bounding all of the actual values. The worst case relative error for the initial parameter estimate was 19%, which is more than satisfactory for an initial value to be used in an estimation algorithm. The relationships developed in this section form an important building block for subsequent sections.

8.4 Winding Resistance

As part of a routine test schedule for a transformer, winding resistance is a typical measurement [53]. Due to the large copper cross sectional areas of the conductors, a power transformer winding will have a resistance that is relatively small. Under these circumstances, to obtain an accurate measurement it is necessary to use a precision instrument, e.g. a Wheatstone bridge.

When the winding under test is of a star connection topology, the measurement can be taken between two of the terminals with the other terminals left open circuit, see Figure 8.1(a). The resistance measurement is effectively taken across two windings in series,

$$R_W = \frac{R_{TT}}{2} \ \Omega \ , \tag{8.10}$$

where R_W is the winding resistance for an individual phase and R_{TT} is the measured resistance between two terminals.

For the case of a delta connected winding, see Figure 8.1(b), there is a series and parallel combination to be considered. In this case it is assumed that all windings are identical in terms of their resistance, hence,

$$R_{TT} = \frac{R_W \cdot (R_W + R_W)}{R_W + R_W + R_W} .$$
(8.11)

Rearranging (8.11) in terms of the winding resistance R_W ,

$$R_W = \frac{3R_{TT}}{2} \ \Omega \ . \tag{8.12}$$

As in the previous section, a generic formulation of (8.10) and (8.12) can be made through the use of the connection scaling factor κ , where,

$$R_W = \frac{\kappa^2 R_{TT}}{2} \ \Omega \ . \tag{8.13}$$

The winding resistance can be used to determine the DC winding resistance term in the transformer model. By incorporating the cross sectional area estimate from Section 8.3, constrained estimates for the skin effect can also be realised. In addition, the winding resistance is used to estimate the winding conductor length which in turn is used to determine the number of HV winding turns .

8.5 Conductor Length

The copper and associated alloys that are used in power transformer manufacture have a conductivity of approximately $\sigma = 58 \times 10^6$ [53]. As is well known, the resistance of a



Figure 8.1: Measurement of winding resistance utilising a Wheatstone bridge (WB); (a) Star connection, (b) Delta connection

conductor of length l and cross sectional area A can be defined as,

.

$$R = \frac{l}{\sigma A} \quad . \tag{8.14}$$

By rearranging (8.14) together with the relationships derived for cross sectional area in Section 8.3 and resistance in Section 8.4, the constraints for the winding conductor length can be determined,

$$[l_{CL}, l_{CU}] = \sigma R_W[A_{CL}, A_{CU}] \times 10^{-6} \text{ m}$$

$$= \sigma \left(\frac{\kappa^2 R_{TT}}{2}\right) \cdot \frac{i_{HR}}{2\kappa} \left[\frac{1}{2}, 1\right] \times 10^{-6} \text{ m}$$

$$[l_{CL}, l_{CU}] = \frac{\sigma \kappa R_{TT} i_{HR}}{4} \left[\frac{1}{2}, 1\right] \times 10^{-6} \text{ m} . \qquad (8.15)$$

In (8.15) l_{CL} and l_{CU} represent the lower and upper bounds respectively, and i_{HR} is the rated current for the HV winding.

The initial parameter estimate for the winding conductor length is taken as the midpoint between the constraints,

$$\hat{l}_C = \frac{3\sigma\kappa R_{TT} i_{HR}}{16} \times 10^{-6} \text{ m} .$$
 (8.16)

The initial parameter estimate and constraints for the winding conductor length, (8.16) and (8.15) respectively, are demonstrated in the following example.

Transformer Description	Act.	LB	Est.	UB	Err. (%)
18MVA 33kV/6.6kV Dyn11	1444	906	1359	1812	6
15MVA 33kV/11kV Dyn11	1572	953	1429	1906	9
5MVA 66kV/11kV Dyn11	3595	1953	2930	3906	19
1.3MVA 11kV/433V Dyn1	1208	839	1258	1678	4
200kVA 11kV/433kV Dyn11	1610	940	1410	1880	12

Table 8.2: High voltage winding conductor length for several power transformers. Dimensions are in metres.

Example : Calculation of the initial parameter estimate and constraints for the conductor length of the high voltage winding of a Dyn1 1.3MVA 11kV/433V distribution transformer.

To determine the constraints for the high voltage winding conductor length of the transformer, reference is made to (8.15). Since the high voltage winding is delta connected, $\kappa = \sqrt{3}$. The current rating is directly obtained from the nameplate details, $i_{HR} = 73A$. Finally, the resistance is accurately measured between two high voltage terminals using a Yokogawa wheatstone bridge. The results obtained were, 0.92 Ω between the A and B terminals, and 0.91 Ω between the B and C terminals. Taking the mean resistance together with the rated current and substituting into (8.15),

$$[l_{CL}, l_{CU}]_{HV} = \frac{58 \times 10^6 \times \sqrt{3} \times 0.915 \times 73}{4} \left[\frac{1}{2}, 1\right] \times 10^{-6} \text{ m}$$

= [839, 1678] m . (8.17)

The corresponding initial parameter estimate is,

$$\hat{l}_C = \frac{839 + 1678}{2}$$

= 1258 m . (8.18)

The actual winding conductor length is 1208 metres which is within the constraints [839, 1678]. The initial parameter estimate of 1258 metres is only in error by 4% relative to the actual winding conductor length.

Table 8.2 presents the high voltage winding conductor lengths of a range of power transformers. In all cases, the conductor lengths are within the calculated constraints, UB and LB respectively. The worst case relative error for the initial parameter estimate is 19%.

The initial parameter estimate and constraints developed in this section utilise information that can readily be obtained on site, and as such, are quite useful for modelling purposes. These relationships are fundamental to Section 8.7 which quantifies the number of turns in the HV and LV windings.

8.6 Mean Diameter of the High Voltage Winding

It is proposed that an approximation for the mean diameter of the high voltage winding can be obtained by utilising the transformer external tank dimensions. This is achieved by taking advantage of the "approximately" symmetrical structure of a power transformer relative to the overall tank dimensions. A cross sectional top view of the Dyn1 1.3MVA 11kV/433V distribution transformer, which has been used as an example in previous sections, is presented in Figure 8.2. All of the dimensioned items in this figure are to scale.

It can be observed in Figure 8.2, that the structural layout often has many symmetrical features. In addition, it is noted that the high voltage winding is generally the outside winding in power transformer construction. The feature exploited here is the relative similarity between the high voltage winding outside diameter, and the corresponding distance to the tank end and side walls (Δ_{EW} and Δ_{SW} respectively). Though these distances may vary considerably due to tap changer position, bus bar routing and other construction features, relative to the overall tank dimensions, $l_{TL} \times l_{TW}$, the four dimensions can be considered to be proportionally similar.

Assumption 1. From the above discussion a generic high voltage winding to tank wall distance of Δ_W is assumed,

$$\Delta_W \approx \Delta_{EW} \approx \Delta_{SW} \quad . \tag{8.19}$$

From Figure 8.2 and with reference to (8.19), it can be observed that the overall length of the tank, l_{TL} , can be decomposed into the following terms,

$$l_{TL} \approx 2\Delta_W + 3\Theta_{HOD} + 2\delta_{HV} , \qquad (8.20)$$

where Δ_W is the distance between the tank end wall and the high voltage winding outside diameter, Θ_{HOD} is the high voltage winding outside diameter, and δ_{HV} is the clearance distance between two adjacent high voltage windings.

Assumption 2. The overall length of the transformer tank, l_{TL} , is much greater than the distance between two adjacent high voltage windings, δ_{HV} ,

$$l_{TL} \gg 2\delta_{HV} \quad . \tag{8.21}$$



Figure 8.2: Top view cross section of a Dyn1 $1.3 \rm MVA~11 kV/433V$ distribution transformer. Dimensioned items are to scale.

Therefore the overall tank length relationship, (8.20), can be rewritten as,

$$l_{TL} \approx 2\Delta_W + 3\Theta_{HOD} \quad . \tag{8.22}$$

Similarly, as can be seen in Figure 8.2 and with reference to (8.19), the overall tank width can be decomposed into the following terms,

$$l_{TW} \approx 2\Delta_W + \Theta_{HOD} \quad , \tag{8.23}$$

where in this case Δ_W is the distance between the tank side wall and the high voltage winding outside diameter. Subtracting (8.23) from (8.22) gives,

$$l_{TL} - l_{TW} \approx 2\Theta_{HOD} \quad . \tag{8.24}$$

Rearranging (8.24),

$$\Theta_{HOD} \approx \frac{l_{TL} - l_{TW}}{2} \quad . \tag{8.25}$$

The diameter of a high voltage winding turn is dependent upon its position within the layer of an individual disc. With reference to Figure 8.2, it is observed that the variation between the inside and outside diameter of the high voltage winding disc is given by α_{HV} . Therefore, the mean diameter of a high voltage disc, Θ_H , can be considered to be,

$$\Theta_H = \Theta_{HOD} - \alpha_{HV} \quad . \tag{8.26}$$

Assumption 3. Since the outside diameter is typically much larger than the diameter variation, α_{HV} , this work assumes that the outside diameter of the high voltage disc approximates the mean diameter,

$$\Theta_{HOD} \gg \alpha_{HV}$$
 (8.27)

$$\therefore \Theta_H \approx \Theta_{HOD} . \tag{8.28}$$

Equating (8.25) with (8.28),

$$\Theta_H \approx \frac{l_{TL} - l_{TW}}{2} \quad , \tag{8.29}$$

where Θ_H is the mean high voltage winding diameter estimate based on the external tank dimensions of a transformer.

To demonstrate in a practical manner the validity of the assumptions utilised to derive (8.29), the dimensions of two distribution transformers are used as case studies. The two transformers are a 1.3MVA 11kV/433V and a 200kVA 11kV/433V distribution transformer. To accomplish this we first measure and record the dimensions that are

Dimension Reference	1.3MVA 11kV/433V	200kVA 11kV/433V
	1980	1100
l_{TW}	1010	525
l_Y	1090	660
Θ_{HOD}	505	305
$lpha_{HV}$	45	30
δ_{HV}	40	25
Δ_{EW1}	210	100
Δ_{EW2}	210	100
Δ_{SW1}	260	130
Δ_{SW2}	260	130

Table 8.3: Actual 1.3MVA and 200kVA distribution transformer dimensions with respect to Figure 8.2. All dimensions are in millimetres.

specified in Figure 8.2. These values are listed in Table 8.3.

Relative error case study for Assumption 1 :

The first assumption is that the end wall and side wall spacing with respect to the high voltage winding would be proportionally similar with respect to the outer dimensions of the tank, (8.19). In order to quantify the relative error of this approximation, the absolute value of the difference between Δ_{EW} and Δ_{SW} , with respect to the tank width, can be considered,

$$\operatorname{Error} \% = \frac{|\Delta_{SW} - \Delta_{EW}|}{l_{TW}} \times 100 . \qquad (8.30)$$

For the two distribution transformers we have,

Error% (1.3MVA) =
$$\frac{|260 - 210|}{1010} \times 100 = 5\%$$
 (8.31)

Error% (200kVA) =
$$\frac{|130 - 100|}{525} \times 100 = 6\%$$
. (8.32)

It can be observed from (8.31) and (8.32), that the relative error is indeed small and therefore the assumption regarding the similarities of the dimensions is appropriate for these two transformers.

Relative error case study for Assumption 2:

Assumption 2 was $l_{TL} \gg 2\delta_{HV}$, (8.21). The relative error in this case is,

$$\operatorname{Error} \% = \frac{2\delta_{HV}}{l_{TL}} \times 100 . \qquad (8.33)$$

For the two distribution transformers,

Error% (1.3MVA) =
$$\frac{2 \times 40}{1980} \times 100 = 4\%$$
 (8.34)

Error% (200kVA) =
$$\frac{2 \times 25}{1100} \times 100 = 5\%$$
 (8.35)

Once again, the relative error for both transformers based on the assumption that $l_{TL} \gg 2\delta_{HV}$, is small.

Relative error case study for Assumption 3:

Assumption 3 is that the difference between the inner and outer diameters of the high voltage winding, α_{HV} , is small relative to the outer diameter value, Θ_{HOD} (8.27),

$$\operatorname{Error} \% = \frac{\alpha_{HV}}{\Theta_{HOD}} \times 100 . \tag{8.36}$$

For the two distribution transformers,

Error% (1.3MVA) =
$$\frac{45}{505} \times 100 = 9\%$$
 (8.37)

Error% (200kVA) =
$$\frac{30}{305} \times 100 = 10\%$$
 (8.38)

The assumption that α_{HV} is small when compared to the outer diameter leads to relative errors for the 1.3MVA transformer of 9% and the 200kVA transformer of 10%. These errors are reasonably small and represent an acceptable approximation for the intended application.

It has been demonstrated for the two transformers in the case studies that the assumptions made to determine Θ_H resulted in only small relative errors. To accommodate the broader spectrum of transformer designs, it is proposed that constraints based on $\pm 20\%$ be defined. Hence from (8.29), constraints for the mean winding diameter are proposed to be,

$$[\Theta_{HL}, \Theta_{HU}] = \frac{l_{TL} - l_{TW}}{2} [80\%, 120\%] = \frac{l_{TL} - l_{TW}}{5} [2, 3] \text{ m} , \qquad (8.39)$$

where Θ_{HU} and Θ_{HL} are the upper and lower bounds respectively. The initial parameter

estimate for the mean high voltage winding diameter, from (8.29), is,

$$\hat{\Theta}_H = \frac{l_{TL} - l_{TW}}{2} \,\mathrm{m} \,.$$
 (8.40)

The initial parameter estimate and constraints of (8.40) and (8.39) are verified in the following example.

Example : Calculation of the initial parameter estimate and constraints for the mean diameter of the high voltage winding of a Dyn1 1.3MVA 11kV/433V distribution transformer.

From (8.40) and with reference to Table 8.3 for the 1.3MVA transformer,

$$\hat{\Theta}_H = \frac{l_{TL} - l_{TW}}{2} = \frac{1980 - 1010}{2} = 485 \text{ mm}$$
 (8.41)

From (8.39), the parameter constraints are,

$$[\Theta_{HL}, \Theta_{HU}] = [388, 582] \text{ mm}$$

Hence, based on the transformer tank dimensions, the initial parameter estimate for the mean high voltage winding diameter is 485mm with constraints of [388,582]. The mean diameter using actual measurements is,

$$\Theta_H = \Theta_{HOD} - \alpha_{HV} = 505 - 45 = 460 \text{ mm.}$$
(8.42)

This results in a relative error of 5% which is well within the specified constraints.

Table 8.4 compares the actual mean high voltage winding diameter to the calculated initial parameter estimate and constraints of a range of power transformers. The table also lists the relative error between the actual mean diameter and the estimated value (column Err.). The largest relative error is just 11% with all cases well within their constraints, the practical applicability of this technique has been verified.

This section has proposed an approximation to the mean high voltage winding diameter using only the external dimensions of the transformer tank. The validity of the proposed relationships has been demonstrated on a variety of power transformers. The results considering the nature of the approximation are very good. It is acknowledged that these relationships will not always be applicable, however, in many cases it will facilitate a useful and non-intrusive initial approximation of the mean high voltage winding diameter.

Transformer Description	Act.	LB	Est.	UB	Err. (%)
18MVA 33kV/6.6kV Dyn11	701	624	780	936	11
15MVA 33kV/11kV Dyn11	664	540	675	810	2
5MVA 66kV/11kV Dyn11	636	512	640	768	1
1.3MVA 11kV/433V Dyn1	460	388	485	582	5
200kVA 11kV/433kV Dyn11	275	230	288	345	5

Table 8.4: Mean high voltage winding diameter for several power transformers. Dimensions are in millimetres.

8.7 Number of High Voltage Winding Turns

Utilising the winding cross sectional area constraints from Section 8.3, together with an accurate measurement of the winding resistance, constraints for the length of the winding conductor were obtained in Section 8.5. In addition, a relationship to determine an approximation for the mean diameter of the high voltage winding was formulated in Section 8.6. By combining these results, it is possible to derive constraints for the number of turns in the high voltage winding.

The number of turns in a high voltage winding, N_H , can be approximated by the total winding conductor length, l_H , divided by the circumference, $\pi \Theta_H$, where Θ_H is the mean high voltage winding diameter. Hence the number of high voltage winding turns is,

$$N_H = \frac{l_H}{\pi \Theta_H} \quad . \tag{8.43}$$

Substituting (8.15) and (8.39) into (8.43) for the corresponding winding conductor length, l_H , and mean winding diameter, Θ_H , constraints for the number of turns in the high voltage winding are given by,

$$[N_{HL}, N_{HU}] = \left(\frac{\sigma \kappa_H R_{HTT} i_{HR}}{4} \left[\frac{1}{2}, 1\right] \times 10^{-6}\right) \cdot \left(\pi \frac{\left(l_{TL} - l_{TW}\right)}{5} [2, 3]\right)^{-1}$$
$$= \frac{5\sigma \kappa_H R_{HTT} i_{HR} \left[\frac{1}{2}, 1\right]}{4\pi \left(l_{TL} - l_{TW}\right) \left[2, 3\right]} \times 10^{-6}$$
$$\therefore [N_{HL}, N_{HU}] = \frac{5\sigma \kappa_H R_{HTT} i_{HR}}{24\pi \left(l_{TL} - l_{TW}\right)} \left[1, 3\right] \times 10^{-6} \text{ turns }.$$
(8.44)

In (8.44), the upper and lower bounds are N_{HL} and N_{HU} respectively. In addition, κ_H is the high voltage winding connection scaling factor (8.3), R_{HTT} is the high voltage winding inter-terminal resistance and i_{HR} is the high voltage terminal current rating.

Using the constraint midpoint from (8.44) as the initial parameter estimate for the

number of HV winding turns, N_H ,

$$\hat{N}_H = \frac{5\sigma\kappa_H R_{HTT} i_{HR}}{12\pi \left(l_{TL} - l_{TW} \right)} \times 10^{-6} \text{ turns }.$$
(8.45)

In order to demonstrate the applicability of (8.44) and (8.45), an example follows.

Example : Calculation of the initial parameter estimate and constraints for the number of turns in the high voltage winding of a Dyn1 1.3MVA 11kV/433V distribution transformer.

The typical value for the conductor conductivity is $\sigma = 58 \times 10^6$ MS/m [53]. From the transformer's nameplate and external measurements, the following details are obtained:

- 1. Vector group is Dyn1 \Rightarrow High voltage winding is in delta $\Rightarrow \kappa_H = \sqrt{3}$
- 2. Resistance between high voltage terminals $\Rightarrow R_{HTT} = 0.915 \Omega$
- 3. Nameplate rated HV current $\Rightarrow i_{HR} = 73 \text{ A}$
- 4. Transformer tank length $\Rightarrow l_{TL} = 1980 \,\mathrm{mm}$
- 5. Transformer tank width $\Rightarrow l_{TW} = 1010 \text{ mm}$

Substituting these values into (8.44),

$$[N_{HL}, N_{HU}] = \frac{5 \times 58 \times 10^6 \times \sqrt{3} \times 0.915 \times 73}{24\pi \left(1980 - 1010\right) \times 10^{-3}} \left[1, 3\right] \times 10^{-6} \text{ turns}$$

which results in the HV winding turn constraints of,

$$[N_{HL}, N_{HU}] = [459, 1376] \text{ turns} . \tag{8.46}$$

The initial parameter estimate, \hat{N}_H , is the midpoint of (8.46),

$$N_H \approx \frac{N_{HU} + N_{HL}}{2} = \frac{1376 + 459}{2} = 917$$
 (8.47)

The actual number of transformer turns is 852 which is within the constraints of (8.46). The initial parameter estimate for the number of turns, from (8.47), is 917. The relative error of the initial parameter estimate to the actual number of turns is 8%.

This approach is now applied to a range of power transformers with the results presented in Table 8.5. From the table it is clear that the actual number of turns is within the limits in each case. The initial parameter estimate had a worst case relative error of just 10%.

Transformer Description	Act.	LB	Est.	UB	Err. (%)
18MVA 33kV/6.6kV Dyn11	656	308	616	925	6
15MVA 33kV/11kV Dyn11	754	375	749	1124	1
5MVA 66kV/11kV Dyn11	1800	810	1620	2430	10
$1.3\mathrm{MVA}~11\mathrm{kV}/433\mathrm{V}~\mathrm{Dyn1}$	852	459	917	1377	8
200kVA 11kV/433kV Dyn11	1864	868	1736	2603	7

Table 8.5: The number of high voltage winding turns for each phase of a range of power transformers of varying size.

This section has derived relationships to determine the initial parameter estimate and constraints for the number of turns in the high voltage winding of a power transformer. These approximations relied upon nameplate parameters and external measurements only. The approach has been verified against a range of power transformers.

8.8 Number of Low Voltage Winding Turns

In Section 8.7, relationships were derived in order to determine an initial parameter estimate and constraints for the number of turns in a transformer's HV winding. Using these relationships, together with the transformer's vector group and nameplate voltage ratings, similar relationships can be derived for the number of turns in a transformer's LV winding. In Section 2.9.1 the transformer's voltage gain with respect to the transformer turns ratio, \bar{a} , was derived for each vector group. The results were provided in Table 2.3.

From Table 2.3, both the Dd and Yy vector groups have a turns ratio of,

$$\bar{a} = \frac{v_{HR}}{v_{LR}} , \qquad (8.48)$$

where v_{HR} and v_{LR} are the nameplate voltage ratings for the transformer's high and low voltage sides respectively.

For the Dy vector group, from Table 2.3, the turns ratio is given by,

$$\bar{a} = \frac{\sqrt{3}v_{HR}}{v_{LR}} . \tag{8.49}$$

For the Yd vector group, the turns ratio is given by,

$$\bar{a} = \frac{v_{HR}}{\sqrt{3}v_{LR}} . \tag{8.50}$$

The transformer turns ratios, (8.48), (8.49) and (8.50), are defined for each vector

Vector Group	Transformer Turns Ratio (\bar{a})
Dd	$rac{v_{HR}}{v_{LR}}$
Dy	$rac{\sqrt{3}v_{HR}}{v_{LR}}$
Yd	$rac{v_{HR}}{\sqrt{3}v_{LR}}$
Yy	$\frac{v_{HR}}{v_{LR}}$

Table 8.6: Vector group relationship with the transformer turns ratio

group in Table 8.6. Rearranging the transformer turns ratio, (2.5), in terms of N_L ,

$$N_L = \frac{N_H}{\bar{a}} \quad . \tag{8.51}$$

The equations (8.48), (8.49) and (8.50) are the ideal case. For the turns ratio the assumption is generally made that the applied terminal voltage is equivalent to the magnetising electromotive force [51], that is, with reference to Figure 2.14, $v_X \approx e_{mX}$. This approximation is appropriate as it is understood that the no load current of a transformer is very small relative to its full load current (0.2 to 2%) [71]. The small error introduced by this assumption is satisfactorily encompassed within the constraints of N_H defined in (8.44). On this basis, with reference to (8.44) and (8.51), the constraints for the number of turns in the low voltage winding is,

$$[N_{LL}, N_{LU}] = \frac{[N_{HL}, N_{HU}]}{\bar{a}}$$
$$= \frac{5\sigma\kappa_H R_{HTT} i_{HR}}{24\bar{a}\pi \left(l_{TL} - l_{TW}\right)} \left[1, 3\right] \times 10^{-6} \text{ turns }.$$
(8.52)

The corresponding initial parameter estimate \hat{N}_L is given by,

$$\hat{N}_L = \frac{5\sigma\kappa_H R_{HTT} i_{HR}}{12\bar{a}\pi \left(l_{TL} - l_{TW}\right)} \times 10^{-6} \text{ turns }, \qquad (8.53)$$

An example follows 1 .

Example : Calculation of the initial parameter estimate and constraints for the number of turns in the low voltage winding of an 18MVA 33kV/6.6kV Dyn11 power transformer.

The constraints for the number of turns in the low voltage winding of the 18MVA transformer can be determined with reference to the high voltage winding equivalent

¹The 1.3MVA transformer used in previous examples cannot be used in this section since the exact number of low voltage turns cannot be accurately determined. The manufacturer provided the number of turns for the 18MVA transformer used in this example.

in Table 8.5 and the Dy connection voltage ratio of Table 8.6. The resulting constraints are,

$$[N_{LL}, N_{LU}] = \frac{6600}{\sqrt{3} \times 33000} . [308, 925]$$

= [36, 107] turns . (8.54)

The initial parameter estimate for the number of low voltage winding turns can be made with reference to the corresponding high voltage initial parameter estimate in Table 8.5, and the Dy connection voltage ratio of Table 8.6,

$$\hat{N}_{L} = \frac{616 \times 6600}{\sqrt{3} \times 33000} \\
= 71 \text{ turns} .$$
(8.55)

Hence the low voltage winding of the 18MVA transformer has bounds of between 36 and 107 turns, with an initial parameter estimate of 71 turns. The actual number of turns for the low voltage winding is 72. This is within the specified constraints, (8.54), and the initial parameter estimate of 71 turns is in error by less than 2%.

This section has obtained relationships for the initial parameter estimate and constraints for the number of turns of the low voltage winding of a power transformer. As with other relationships in this chapter, only nameplate details and external dimensions are utilised.

8.9 Transformer Core Linear Dimensions

A parameter that is integral to the development of a geometrically based transformer model is the reluctance of the transformer's magnetic core. Reluctance is proportional to its path length, l, and inversely proportional to its cross sectional area, A. This can be expressed mathematically as,

$$\mathcal{R} = \frac{l}{\mu A} \tag{8.56}$$

where μ is the permeability of the core material. A detailed investigation of permeability is described in Chapter 3.

In this section we are interested in estimating the reluctance path length l. Figure 8.3 presents a three phase, three limb core. Typically the cross sectional area of the yoke and limbs of a three limb core is the same [53]. Current research has tended to focus on placing limits on the yoke to limb ratio. In the paper by de Leon [34], a single phase transformer yoke to limb ratio for a "tall" transformer was 1:1 and a "short" transformer was 4:1. However, this research proposes tighter limits by taking into account common



Figure 8.3: Simplified drawing of the magnetic core of a three limb core form transformer

design practice and external tank dimensions.

8.9.1 Mean Length of the Core Yoke

With reference to Figure 8.2, it can be observed that the mean yoke length, l_Y , is equivalent to the distance between the two outside winding centres, i.e.

$$l_Y = 2\Theta_{HOD} + 2\delta_{HV} \ . \tag{8.57}$$

In practice $\Theta_{HOD} \gg \delta_{HV}$ (for the two distribution transformers whose dimensions are listed in Table 8.3 the difference is larger than an order of magnitude). Hence a reasonable approximation for the mean yoke length is,

$$l_Y \approx 2\Theta_{HOD}$$
 . (8.58)

By substituting the high voltage winding diameter approximation from (8.40) into (8.58), an estimate for the core yoke length, \hat{l}_Y , can be defined in terms of the external dimensions of the transformer tank,

$$\hat{l}_Y = l_{TL} - l_{TW} \quad . \tag{8.59}$$

To set appropriate constraints, it is proposed that the limits for the mean yoke length be based on $\pm 20\%$ of the parameter estimate as per the relationships in Section 8.6. As such, the mean yoke length constraints are defined as,

$$[l_{YL}, l_{YU}] = (l_{TL} - l_{TW}) [0.8, 1.2] .$$
(8.60)

Transformer Description	Act.	LB	Est.	UB	Err. (%)
18MVA 33kV/6.6kV Dyn11	1590	1248	1560	1872	2
15MVA 33 kV $/11$ kV Dyn11	1500	1080	1350	1620	10
5MVA 66kV/11kV Dyn11	1440	1024	1280	1536	11
1.3MVA 11kV/433V Dyn1	1090	776	970	1164	11
200kVA 11kV/433kV Dyn11	660	460	575	690	13

Table 8.7: Transformer core mean yoke length for several power transformers. Dimensions are in millimetres.

The initial parameter estimate and constraints for the mean yoke length of the transformer core, (8.59) and (8.60), are obtained by simply finding the difference between the transformer tank length and width. An example follows.

Example : Calculation of the initial parameter estimate and constraints for the mean yoke length of a Dyn1 1.3MVA 11kV/433V distribution transformer core.

An initial parameter estimate for the mean yoke length, (8.59), requires the external length and width of the transformer tank. With reference to Table 8.3,

$$\hat{l}_Y = l_{TL} - l_{TW} = 1980 - 1010 = 970 \text{ mm}$$
 (8.61)

The constraints, (8.60), are then given by,

$$[l_{YL}, l_{YU}] = 970 [0.8, 1.2] = [776, 1164] \text{ mm} . \tag{8.62}$$

With reference to Figure 8.2 and Table 8.3, the actual mean yoke length l_Y is 1090mm. This is within the bounds of (8.62). The initial parameter estimate of 970mm is within 11% of the actual value.

This approach is applied to a range of power transformers with the results presented in Table 8.7. The constraints encapsulate the actual yoke length in all cases with a maximum initial parameter estimate error of just 13%.

8.9.2 Mean Length of the Core Limb

Estimating the mean limb length is not as straightforward as in the case of the mean yoke length. Previous sections in this chapter utilised the symmetry of the transformer to estimate internal dimensions. This is not possible in a generic sense when considering the core limb length. It is quite common for a power transformer to have tap changer



Figure 8.4: Side view cross section of a transformer. Based on a Dyn1 1.3MVA 11kV/433V distribution transformer. Dimensioned items are to scale.

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Dimension Reference	$1.3\mathrm{MVA}~11\mathrm{kV}/433\mathrm{V}$	$200 \mathrm{kVA}~11 \mathrm{kV}/433 \mathrm{V}$
l_{TW}	1010	525
l_{TH}	2170	1100
l_E	1235	620
Θ_{COD}	215	150
Δ_{TW1}	830	500
Δ_{TW2}	520	185
Δ_{SW1}	260	130
Δ_{SW2}	260	130

Table 8.8: 1.3MVA 11kV/433V and 200kVA 11kV/433V distribution transformer reference dimensions with respect to Figures 8.2 and 8.4. Dimensions in millimeters.

and connection bus bars positioned above the windings, removing the possibility of any vertical symmetry that could be used to determine the limb length of the core. A scaled drawing of the 1.3MVA 11kV/433V distribution transformer is given in Figure 8.4 as an example. Table 8.8 lists the reference dimensions (including those of a 200kVA 11kV/433 transformer) for Figure 8.4.

Not being able to utilise structural symmetry makes it more difficult to obtain an approximation for the mean limb length, l_E . Some insight is obtained however, by investigating the implications of the height of a transformer tank, l_{TH} . It is obvious that the mean core limb height must be significantly less than the tank height. By taking into account typical clearance distances, bus bar connections and possibly tap changing equipment mounted in the upper chamber of the tank, it is proposed that an upper bound for the mean core limb height, l_E , could be set to 80% of the tank height,

$$l_{EU} = 0.8 l_{TH} {.} {(8.63)}$$

This is supported empirically by Table 8.9. Furthermore, the core and windings occupy the majority of the space within a transformer tank. It is improbable for the tank to be constructed with an absolute limb height that was less than 50% of the tank height. Therefore, taking into account that the mean height is less than the absolute height, it is proposed that an appropriate lower bound could be set to,

$$l_{EL} = 0.4 l_{TH} \quad . \tag{8.64}$$

This is also supported empirically by Table 8.9. On this basis, constraints are proposed that leverage knowledge of the height of the transformer tank and typical construction practices. The mean limb length constraints are then,

$$[l_{EL}, l_{EU}] = l_{TH} [0.4, 0.8] \tag{8.65}$$

From (8.65), an appropriate initial parameter estimate for the mean limb length would simply be the mid point value of the constraints,

$$\hat{l}_E = 0.6 l_{TH}$$
 . (8.66)

To demonstrate the mean limb length defined in (8.65) and (8.66), an example follows.

Example : Calculation of the initial parameter estimate and constraints for the mean limb length of a Dyn1 1.3MVA 11kV/433V distribution transformer core.

With reference to Figure 8.4 and Table 8.8, the mean limb length constraints, (8.65), for the 1.3MVA transformer are,

$$[l_{EL}, l_{EU}] = l_{TH} [0.4, 0.8] = [868, 1736] \text{ mm} .$$
(8.67)

The mean limb length initial parameter estimate from (8.66) is,

$$\hat{l}_E = 0.6 l_{TH} = 0.6 \times 2170 = 1302 \text{ mm}$$
 (8.68)

The actual mean limb length, \hat{l}_E , is 1235mm. This is within the constraints of (8.67), and the initial parameter estimate of 1302mm is within 5%.

Application of the mean limb length relationships of (8.65) and (8.66) to a range of power transformers is presented in Table 8.9. In all cases the transformer core limb lengths are within the prescribed constraints. The largest relative error for the initial parameter estimate was 17%, however in two cases, the relative error was less than 0.5%.

This section has developed relationships for approximating the mean core yoke and limb lengths of a power transformer's magnetic circuit. These relationships rely solely on the external dimensions of the power transformer's tank. This is accomplished by taking advantage of intrinsic design properties and associated structural symmetry. When applied to a range of power transformers of varying size, the largest initial parameter estimate error for both the yoke and limb lengths, was 17%. On this basis it is proposed that this approach is a significant advance over the reliance on a broad yoke to limb ratio range [34] that is currently employed in research today.

Transformer Description	Act.	LB	Est.	UB	Err. (%)
18MVA 33kV/6.6kV Dyn11	1800	1204	1806	2408	0
15MVA 33kV/11kV Dyn11	1950	1080	1620	2160	-17
5MVA 66kV/11kV Dyn11	1610	1078	1617	2156	0
1.3MVA 11kV/433V Dyn1	1235	868	1302	1736	5
200kVA 11kV/433kV Dyn11	620	440	660	880	6

Table 8.9: Transformer core mean limb length for several power transformers. Dimensions are in millimetres.

8.10 Cross Sectional Area of a Transformer core

As discussed in Section 8.9, to determine the reluctance of a power transformer's core, it is necessary to determine its cross sectional area. This section derives the initial parameter estimate and constraints for the cross sectional area of the core. The derivation utilises results from previous sections as well as general transformer design principles. As with the derivations from previous sections, the relationships do not require internal structural knowledge of the individual transformer.

Faraday's law states that the induced voltage is proportional to the time rate of change of flux linkage. From Lenz's law, this induced voltage must have a polarity that opposes a change in the flux linkage. Mathematically this can be expressed as,

$$e_m = -N \frac{d(\Phi_m)}{dt} \quad , \tag{8.69}$$

where e_m is the induced voltage, Φ_m the magnetising flux and N the number of winding turns. When the voltage is sinusoidal, it can be shown [53] that (8.69) can be expressed in terms of volts per turn, i.e.

$$\frac{e_m}{N} = 4.44 f \Phi_m \quad \text{V/turn} \quad , \tag{8.70}$$

where f is the rated frequency of operation in Hertz of the transformer. It can be more convenient to express (8.70) in terms of the maximum flux density B_{max} with respect to the cross sectional area of the core, A_{CS} ,

$$\frac{e_m}{N} = 4.44 B_{max} A_{CS} f \quad \text{V/turn} \quad . \tag{8.71}$$

Equation (8.71) is an important relationship used by transformer designers. It allows the designer to determine the output volts per turn based on the peak flux density, core cross sectional area and operating frequency. In this section (8.71) plays an important role in facilitating the approximation of the cross sectional area of the core.

Typically, modern power transformers use cold rolled grain oriented silicon steel



Figure 8.5: B-H Curve for 3.5% silicon electrical steel at 50Hz. (a) Full B-H curve (log scale). (b) B-H curve highlighting the typical peak operating flux region of a power transformer (linear scale).

(CRGO) in the manufacture of the core's laminations. The saturation flux density for CRGO silicon steel is generally 2.0 T [71]. The point where the B-H curve begins to saturate is generally referred to as the "knee". The transformer manufacturer must design the peak operating flux density to be less than this value due to the significantly greater magnetising currents required for incremental increases in flux density as saturation is approached. However, having an operating flux density that is well below the "knee", does not utilise all of the core's potential, ie, it has too much "iron", and is therefore not cost effective from a manufacturing perspective. The most effective design is to position the maximum operating flux density just below the "knee".

Over excitation of a transformer core can occur with system over-voltage. It is not uncommon for an electrical system to experience a continuous over-voltage as high as 10% [53]. The resulting over excitation will lead to an increase in core losses and a resulting increase in temperature. System losses like these should be avoided where possible as the prolonged increase in operating temperature of a transformer will result in a shortened operating life [83]. As a consequence, a guideline [71] was proposed which is based on the voltage profile of the transformer application,

$$B_{MP} = \frac{1.9}{(1+a\%)} \ \mathrm{T} \ , \tag{8.72}$$

where B_{MP} is the peak operating flux density and a% is a factor dependent on the voltage profile. Based on this guideline, for a system that has a relatively constant voltage profile, a small over excitation factor of 5% could be nominated relative to a 1.9T upper bound. From (8.72),

$$B_{MP} = \frac{1.9}{(1+5\%)} = 1.8 \,\mathrm{T} \quad . \tag{8.73}$$

Hence giving a nominal peak operating flux of 1.8T. However, in the case where the voltage profile of a power network was not as constant, a factor of 15% may be nominated which would result in a peak operating flux of 1.65T. This lower peak operating flux facilitates a greater tolerance for over-voltage conditions. Typical peak operating flux densities range between 1.6 and 1.8T for CRGO silicon steel [53, 20]. This nominal range is depicted in the B-H curve for 3.5% silicon electrical steel at 50Hz, Figure 8.5.

By coupling the knowledge of the typical peak operating flux density range, with the volt/turn relationship from (8.71), we have,

$$\frac{e_m}{N} = 4.44 A_{CS} f [1.6, 1.8] \text{ V/turn} ,$$
 (8.74)

where [1.6, 1.8] are the peak flux density constraints. The frequency is a known parameter and is regionally dependent. Rearranging (8.74) in terms of cross sectional area gives

$$A_{CS} = \frac{e_m}{4.44 f N \left[1.6, 1.8\right]} \, \mathrm{m}^2 \, . \tag{8.75}$$

As discussed in Section 8.8, since the no load current of a transformer is small relative to its full load current (0.2 to 2%), the applied terminal voltage, V, can be assumed to be approximately equal to the magnetising electromotive force,

$$A_{CS} = \frac{V}{4.44fN[1.6, 1.8]} = \frac{V[0.125, 0.141]}{fN} \text{ m}^2 .$$
(8.76)

The volts per turn relationship is dependent upon the voltage applied to the winding. The winding voltage is dependent upon the transformer connection topology. For a star connection, the winding voltage is phase to neutral and for a delta connection, the winding voltage is phase to phase. However, the transformer nameplate details for voltage are with respect to its phase to phase voltage. As such, a factor of $\sqrt{3}$ is required to be incorporated into (8.76) when referring to the nameplate high voltage rating v_{HR} for a star connection.

Rewriting (8.76) in terms of v_{HR} with respect to the winding connection topology,

$$[A_{CSL}, A_{CSU}] = \begin{cases} \frac{v_{HR}[0.125, 0.141]}{\sqrt{3}fN} & \text{Star connected winding} \\ \\ \frac{v_{HR}[0.125, 0.141]}{fN} & \text{Delta connected winding} \end{cases}$$
(8.77)

From Section 8.7, constraints, (8.44), were developed for the number of turns in the high voltage winding. Restating this relationship for the respective connections and taking into account the connection scaling factor κ , gives

$$[N_{HL}, N_{HU}] = \begin{cases} \frac{5\sigma R_{HTT} i_{HR}}{24\pi \left(l_{TL} - l_{TW}\right)} \left[1, 3\right] \times 10^{-6} & \text{Star connected winding} \\ \\ \frac{5\sqrt{3}\sigma R_{HTT} i_{HR}}{24\pi \left(l_{TL} - l_{TW}\right)} \left[1, 3\right] \times 10^{-6} & \text{Delta connected winding} \end{cases}$$
(8.78)

The number of turns parameter of (8.77), can be replaced by its respective constraints from (8.78). It can be observed that by following this substitution for each connection, a connection independent relationship is obtained,

$$[A_{CSL}, A_{CSU}] = \frac{v_{HR} [0.125, 0.141]}{\sqrt{3}f} \cdot \frac{24\pi (l_{TL} - l_{TW})}{5\sigma R_{HTT} i_{HR} [1, 3] \times 10^{-6}}$$
$$= \frac{8.7 v_{HR} (l_{TL} - l_{TW}) [0.125, 0.141]}{f \sigma R_{HTT} i_{HR} [1, 3]} \times 10^{6}$$
$$= \frac{v_{HR} (l_{TL} - l_{TW}) [0.36, 1.23]}{f \sigma R_{HTT} i_{HR}} \times 10^{6} \text{ m}^{2} , \qquad (8.79)$$

for the constraints on the cross sectional area of the transformer core. Taking the midpoint gives the initial parameter estimate, \hat{A}_{CS} ,

$$\hat{A}_{CS} = \frac{0.8v_{HR} \left(l_{TL} - l_{TW} \right)}{f \sigma R_{HTT} i_{HR}} \times 10^6 \text{ m}^2 .$$
(8.80)

Example : Calculation of the initial parameter estimate and constraints for the cross sectional area of a Dyn1 1.3MVA 11kV/433V distribution transformer core.

The transformer details are taken from the nameplate and external measurements as per previous sections,

1. High voltage rating $\Rightarrow v_{HR} = 11$ kV

Transformer Description	Act.	LB	Est.	UB	Err. (%)
18MVA 33kV/6.6kV Dyn11	0.159	0.089	0.196	0.303	23
15MVA 33kV/11kV Dyn11	0.138	0.073	0.161	0.249	16
5MVA 66kV/11kV Dyn11	0.119	0.067	0.149	0.230	25
1.3MVA 11kV/433V Dyn1	0.036	0.020	0.044	0.068	21
200kVA 11kV/433kV Dyn11	0.018	0.010	0.023	0.036	28

Table 8.10: Transformer core cross sectional area for several power transformers. Dimensions are in m^2 .

- 2. Current rating (high voltage side) $\Rightarrow i_{HR} = 73 \text{ A}$
- 3. Mains frequency $\Rightarrow f = 50$ Hz
- 4. Resistance between high voltage terminals $\Rightarrow R_{HTT} = 0.915 \Omega$
- 5. Transformer tank length $\Rightarrow l_{TL} = 1980 \,\mathrm{mm}$
- 6. Transformer tank width $\Rightarrow l_{TW} = 1010 \,\mathrm{mm}$

Substituting the values into (8.79) to determine the cross sectional area constraints,

$$[A_{CSL}, A_{CSU}] = \frac{11000 \times (1.980 - 1.010) [0.36, 1.23]}{50 \times 58 \times 10^6 \times 0.915 \times 73} \times 10^6$$

= [0.020, 0.068] m². (8.81)

Similarly, from (8.80),

$$\hat{A}_{CS} = \frac{0.8 \times 11000 \times (1.980 - 1.010)}{50 \times 58 \times 10^6 \times 0.915 \times 73} \times 10^6$$

= 0.044 m², (8.82)

where \hat{A}_{CS} is the initial parameter estimate for the cross sectional area of the transformer core. With reference to Figure 8.4 and Table 8.8, the actual cross sectional area is $0.036m^2$ which is within the constraints of (8.81) and is 21% less than the initial parameter estimate value of (8.82).

Application of the cross sectional area relationships of (8.79) and (8.80) to a range of power transformer cores, is presented in Table 8.10. The actual cross sectional area of the transformer core is within the constraints in all cases. The largest relative error for the initial parameter estimate was 28%. Whilst this error is not insignificant, it is more than satisfactory for the initial parameter estimate of a constrained estimation algorithm.

8.11 Conclusion

To be able to use a transformer model to diagnose problems based on condition monitoring data, it is important that the model parameters are physically representative. This can be accomplished by using tight parameter constraints and accurate initial estimates within the estimation algorithm. This will minimise the probability of the solution set converging on a local minima. This chapter has developed relationships for an initial parameter estimate, with corresponding constraints, for several important transformer modelling parameters. Since intellectual property restrictions make access to transformer design documentation rarely possible, these relationships have been developed to use information such as external transformer dimensions, routine test data, and nameplate details. The relationships we obtained were successfully applied to a variety of power transformers ranging in size from 200kVA up to 18MVA ².

 $^{^{2}}$ Due to the demonstrated accuracy of the initial parameter estimates it is proposed that, in the case of similarly sized transformers, there is justification for a further tightening of the constraints. As a consequence, key parameters used in Chapter 9 are constrained to the largest relative error observed by each parameter in this chapter.

Chapter 9

Practical Application of the Transformer Model

9.1 Introduction

The purpose of this chapter is to confirm the veracity of the proposed modelling approach and how it can be used to facilitate the interpretation of FRA. This is achieved by demonstrating that the estimation algorithm accurately determines the value of several key physical parameters without any a priori knowledge of the transformer's internal dimensions. The estimation algorithm is also used to detect subtle changes in key parameter values to demonstrate its potential to assist in FRA interpretation.

This chapter is structured in the following manner. Section 9.2 sets out the procedure undertaken to validate the modelling approach. Section 9.3 discusses the transformer measurements for the prescribed FRA tests. Section 9.4 determines the initial parameter estimate and the corresponding constraints for all of the model parameters used in the estimation algorithm. Section 9.5 details the application of an algorithm to estimate the parameters of the model transfer functions. Section 9.6 details the results and parameter estimates for each of the FRA tests. Section 9.7 demonstrates via a practical example how the estimation algorithm could be used to investigate winding deformation. Concluding remarks are given in Section 9.8.

9.2 Model Validation Procedure

The model validation proceeds in the following manner: The first step is to generate FRA data sets for the HV winding End to End Open Circuit test, the LV Winding End to End Open Circuit test and the Capacitive Interwinding test. The three tests are conducted on each of the three permutations, for example, *Aa*, *Bb*, *Cc* for the Capacitive Interwinding test. This testing schedule results in 9 unique frequency response measurements.

Transfer functions are then created for each of the 9 tests using a seeded parameter



Figure 9.1: High Voltage Winding End to End Open Circuit FRA for a 1.3 MVA $11 \rm kV/433V$ Dyn1 transformer.

set¹. A cost function is then applied that sums all of the residuals that exist between the model transfer function and its respective FRA measurement at each frequency point, for each of the 9 tests. A constrained nonlinear optimisation algorithm² then adjusts the parameter values and repeats the previous step until, ideally, a global cost minimum is obtained. At this point the optimum fitting of the prescribed models to the FRA measurements has been achieved.

Key physical parameters used in the model are then compared against those physically measured from the test transformer. Other key parameters which are not readily quantifiable are compared to FEA and analytical estimates for additional validation.

9.3 Frequency Response Analysis Tests

Three types of FRA tests were conducted on a Dyn1 1.3MVA 11kV/433V distribution transformer, these were the High Voltage Winding End to End Open Circuit test, the Low Voltage Winding End to End Open Circuit test, and the Capacitive Interwinding test. The resulting frequency responses are shown in Figures 9.1, 9.2 and 9.3. For the High Voltage End to End FRA shown in Figure 9.1, the frequency response variation due to inductive disparity is clearly visible when comparing the frequency response for test BA, with tests AC and CB. This is also apparent between test bn and tests an and cn

¹Several of the initial values are based on estimates as defined in Chapter 8

 $^{^{2}}$ Numerical computing software is used to implement the constrained nonlinear optimisation algorithm.



Figure 9.2: Low Voltage Winding End to End Open Circuit FRA for a $1.3\mathrm{MVA}$ $11\mathrm{kV}/433\mathrm{V}$ Dyn1 transformer.



Figure 9.3: Capacitive Interwinding FRA for a 1.3MVA 11kV/433V Dyn1 transformer.

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Description	Value
Vector Group	Dyn1
Cooling	AN/AF
High Voltage Rating	11kV
High Voltage Tap	10.63kV
Low Voltage Rating	433V
HV Current Rating	73A
LV Current Rating	1777A
Ζ %	9.49
Frequency	$50 \mathrm{Hz}$
HV Terminal Resistance	0.915Ω
Tank Length	$1.98\mathrm{m}$
Tank Width	1.01m
Tank Height	2.17m

Table 9.1: Transformer details used to constrain several of the model parameters

in Figure 9.2 and test Cc and tests Aa and Bb of Figure 9.3. Inductive disparity and its effects on Dyn connected transformers is discussed in Appendix A.

9.4 Initial Parameter Estimates

The next step in the procedure is to obtain the initial parameter estimates and corresponding constraints for each of the parameters estimated in the estimation algorithm. Using the relationships developed in Chapter 8, the initial parameter estimates and constraints of several of the model parameters can be determined from the transformer nameplate and external measurements. The external measurements include the transformer tank dimensions and the resistance between the high voltage terminals. The details of the transformer under test are shown in Table 9.1.

The parameters for the transformer model which can be relatively tightly constrained are listed in Table 9.2. This table includes the initial parameter estimates and the corresponding constraints³. The table also contains a reference where details on the parameter and/or the constraints can be found.

Without knowledge of the internal dimensions of the transformer it is difficult to place tight constraints on some of the parameters. As a result, the remaining model parameters, listed in Table 9.3, are given a relatively loose set of constraints.

³The initial parameter estimate is generally assumed to be the midpoint of the constraints.

Parameter	Symbol	Units	LB	Est.	UB	Reference
Initial core perm.	μ_i	${\rm Hm^{-1}}$	150	800	1500	Table 3.1
Core conductivity	σ_C	MSm^{-1}	1.69	1.9	2.1	Sect 3.5.4
Lamination thick.	2b	mm	0.23	0.3	0.35	Sect 2.4.1
Stacking factor	k	-	0.95	0.97	0.99	Sect 3.3.2
Core cross sect.	A_{CS}	m^2	0.031	0.043	0.055	(8.82)
Core yoke length	l_Y	mm	844	970	1096	(8.61)
Core limb length	l_E	mm	1081	1302	1523	(8.68)
HV winding turns	N_X	-	826	917	1009	(8.47)
HV cond. area	A_X	mm^2	12.8	15.8	18.8	Sect.6.4 (8.7)
LV cond. area	A_x	mm^2	444	666	889	Sect.6.4 (8.9)
HV turns per disc	N_{LX}	-	5	25	50	Sect 2.5.2
LV layers	N_{Lx}	-	2	6	10	Sect 2.5.2
HV leak. induct.	L_{LX}	mH	11	39	114	Sect 5.6.5
LV leak. induct.	L_{Lx}	$\mu \mathrm{H}$	2	33	64	Sect 5.6.5
Leak. coeff.	au	-	0	0.5	0.9	Sect 6.3.2
HV-Tank Cap.	C_{gXi}	pF	3	5	11	Sect 6.5.4
HV-HV Cap.	C_{XYi}	pF	5	11	21	Sect 6.5.3

Table 9.2: Tightly constrained transformer parameters.

Parameter	Symbol	Units	LB	Est.	UB	Reference
HV-LV Cap.	C_{Xxi}	pF	10^{-1}	50	10^{+2}	Sect 6.5.1
LV-Core Cap.	C_{gxi}	pF	10^{-1}	50	10^{+2}	Sect 6.5.2
HV Winding Cap.	C_{SXi}	pF	10^{-1}	50	10^{+2}	Sect 6.5.5
LV Winding Cap.	C_{Sxi}	pF	10^{-1}	50	10^{+2}	-
HV-Tank Loss	R_{gXi}	$M\Omega$	10^{-2}	1	10^{+2}	Sect 6.5.4, 6.5.6
HV-LV Loss	R_{Xxi}	$M\Omega$	10^{-2}	1	10^{+2}	Sect 6.5.1, 6.5.6
LV-Core Loss	R_{gxi}	$M\Omega$	10^{-2}	1	10^{+2}	Sect 6.5.2, 6.5.6
HV Winding Loss	R_{SXi}	ΜΩ	10^{-2}	1	10^{+2}	Sect 6.5.5, 6.5.6
LV Winding Loss	R_{Sxi}	ΜΩ	10^{-2}	1	10^{+2}	Sect 6.5.6

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Table 9.3: Loosely constrained transformer parameters.

Note that, in this case, the model utilises 8 sections per phase (n=8). This value of n was based upon a tradeoff between the potential for increased accuracy with a larger value of n, versus the significant increase in computational time required to find the global minima.

9.5 Estimation Algorithm

A constrained nonlinear optimisation algorithm is utilised to estimate the transfer functions based on FRA data. This algorithm determines the best fit between the proposed models and the corresponding FRA data by finding the model parameters that minimise a cost function. The cost, J, is based on the 2-norm between each of the FRA data sets and their corresponding FRA model. The model is applied to each of the three phase permutations for the three types of FRA test, hence J represents the cumulative total of each 2-norm associated with these tests,

$$J = W_{XX} \left(\left\| \log_{10} \left(\frac{\hat{G}_{AC}(jw)}{H_{AC}(jw)} \right) \right\|^{2} + \left\| \log_{10} \left(\frac{\hat{G}_{BA}(jw)}{H_{BA}(jw)} \right) \right\|^{2} + \left\| \log_{10} \left(\frac{\hat{G}_{CB}(jw)}{H_{CB}(jw)} \right) \right\|^{2} \right) \\ W_{Xx} \left(\left\| \log_{10} \left(\frac{\hat{G}_{Aa}(jw)}{H_{Aa}(jw)} \right) \right\|^{2} + \left\| \log_{10} \left(\frac{\hat{G}_{Bb}(jw)}{H_{Bb}(jw)} \right) \right\|^{2} + \left\| \log_{10} \left(\frac{\hat{G}_{Cc}(jw)}{H_{Cc}(jw)} \right) \right\|^{2} \right) \\ W_{xx} \left(\left\| \log_{10} \left(\frac{\hat{G}_{an}(jw)}{H_{an}(jw)} \right) \right\|^{2} + \left\| \log_{10} \left(\frac{\hat{G}_{bn}(jw)}{H_{bn}(jw)} \right) \right\|^{2} + \left\| \log_{10} \left(\frac{\hat{G}_{cn}(jw)}{H_{cn}(jw)} \right) \right\|^{2} \right) \right)$$
(9.1)

Symbol	FRA Test	Weight
W _{XX}	HV End to End	1
W_{Xx}	Capacitive Interwinding	$\frac{1}{5}$
W_{xx}	LV End to End	$\frac{1}{10}$

Table 9.4: Cost function weights for the case study.

In (9.1) $H_{..}(jw)$ represents the FRA data and $\hat{G}_{..}(jw)$ the estimated transfer function for the corresponding FRA model defined in (7.68). Weights can also be given to each type of FRA test. In (9.1) W_{XX} is the weighting given to the HV End to End FRA test, W_{Xx} is the weighting given to the Capacitive Interwinding FRA test, and W_{xx} is the weighting given to the LV End to End FRA test. Due to the dominant influence of the high voltage windings, weights were applied in such a way as to emphasise the HV winding within the cost function. Table 9.4 lists the empirically determined weights given to each of the FRA tests for this example. The resulting estimation algorithm parameter solution set $\hat{\theta}$ is given by,

$$\hat{\theta} = \arg\min\left\{J\right\} \,, \tag{9.2}$$

such that the constraints in Tables 9.2 and 9.3 are satisfied.

9.6 Applying the Model to FRA Data

Using the constrained nonlinear optimisation algorithm described in Section 9.5, each of the 9 transfer functions is fitted to their respective frequency responses. Prior to the acceptance of the estimated parameter solution set $\hat{\theta}$, additional tests were conducted in order to provide a high degree of confidence that the global minima had been determined. The first test was to seed several runs of the original constrained nonlinear optimisation algorithm with uniformly distributed random initial values. The second test loaded $\hat{\theta}$, as well as randomly generated parameter solutions, as initial values into a Simulated Annealing Optimisation Algorithm which employed the same cost function. Every test case supported $\hat{\theta}$ as the global minima.

With $\hat{\theta}$ as the accepted global minima, an analysis of the results for each of the FRA tests is given in the following sections. In addition, since the model parameters are based on physical parameters, an analysis is also conducted on the relative accuracy of the estimated parameters.

9.6.1 High Voltage Winding End to End Open Circuit FRA Test

Bode diagrams for the estimated models of the three High Voltage Winding End to End Open Circuit FRA tests are shown in Figures 9.4 to 9.6. The results are very good both in magnitude and phase for frequencies ≤ 1 MHz. An important result is the estimated



Figure 9.4: Estimated transformer model and the FRA data for the HV Winding End to End Open Circuit test measured between the high voltage A and C terminals on a 1.3MVA 11kV/433V Dyn1 transformer.



Figure 9.5: Estimated transformer model and the FRA data for the HV Winding End to End Open Circuit test measured between the high voltage B and A terminals on a 1.3MVA 11kV/433V Dyn1 transformer.


Figure 9.6: Estimated transformer model and the FRA data for the HV Winding End to End Open Circuit test measured between the high voltage C and B terminals on a 1.3MVA 11kV/433V Dyn1 transformer.



Figure 9.7: Estimated transformer model and the FRA data for the LV Winding End to End Open Circuit test measured between the low voltage terminal and neutral of phase A on a 1.3MVA 11kV/433V Dyn1 transformer.



Figure 9.8: Estimated transformer model and the FRA data for the LV Winding End to End Open Circuit test measured between the low voltage terminal and neutral of phase B on a 1.3MVA 11kV/433V Dyn1 transformer.



Figure 9.9: Estimated transformer model and the FRA data for the LV Winding End to End Open Circuit test measured between the low voltage terminal and neutral of phase C on a 1.3MVA 11kV/433V Dyn1 transformer.



Figure 9.10: Estimated transformer model and the FRA data for the Capacitive Interwinding test measured between the high and low voltage terminals of phase A on a 1.3MVA 11kV/433V Dyn1 transformer.



Figure 9.11: Estimated transformer model and the FRA data for the Capacitive Interwinding test measured between the high and low voltage terminals of phase B on a 1.3MVA 11kV/433V Dyn1 transformer.



Figure 9.12: Estimated transformer model and the FRA data for the Capacitive Interwinding test measured between the high and low voltage terminals of phase C on a 1.3MVA 11kV/433V Dyn1 transformer.

model response to the inductive disparity. Its effect, which can be observed by comparing the significant frequency variation between tests BA and tests AC and CB, is clearly accommodated by the model.

9.6.2 Low Voltage Winding End to End Open Circuit FRA Test

Bode diagrams for the three Low Voltage Winding End to End Open Circuit FRA tests are shown in Figures 9.7 to 9.9. The self resonance for tests *an* and *cn* is at approximately 3.2kHz whereas for test *bn* the self resonance is at approximately 2.6kHz. This is a result of the inductive variation and is emulated by the model. Good fitting results are obtained for frequencies ≤ 1 MHz.

9.6.3 Capacitive Interwinding FRA Test

Bode diagrams for each of the Capacitive Interwinding FRA tests are shown in Figures 9.10 to 9.12. The first resonance for tests Aa and Bb is at approximately 1.5kHz whereas for test Cc the first resonance is at 1.8kHz. This variation is due to inductive disparity and is successfully captured by the model. The overall correlation between the model and the three tests for frequencies ≤ 1 MHz is very good.

9.6. APPLYING THE MODEL TO FRA DATA

Parameter	Symbol	Units	Estimate	Actual	Error
Core cross sectional area	A_{CS}	m^2	0.038	0.036	4%
Core yoke length	l_Y	m	1.1	1.1	0%
Core limb length	l_E	m	1.1	1.2	10%
HV winding turns	N_X	-	912	852	7%

Table 9.5: Comparison between estimated and actual values for key measurable parameters.

Sectional Capacitance	Symbol	Units	Model Estimate	FEA/Theoretical Est.
HV-LV	C_{Xxi}	pF	75	52
LV-Core (Ground)	C_{gxi}	pF	63	44
HV-Tank (Phase A/C)	C_{gXi}	pF	11	6
HV-Tank (Phase B)	C_{gXi}	pF	10	5
HV-HV	C_{XYi}	pF	9	6
HV Winding	C_{SXi}	pF	90	30

Table 9.6: Comparison between the modelled and FEA/theoretical estimates for the sectional capacitance (n = 8).

9.6.4 Confirmation of Parameters

To demonstrate the physical nature of the proposed models, a number of parameters which can be readily measured are compared with their estimated counterparts in Table 9.5. The worst case error was 10%. When considered in the context that the parameters have been determined *without* use of any of the internal dimensions of the transformer, the results are particularly good and demonstrate the physically representative nature of the proposed transformer model.

For the parameters related to capacitance, an accurate physical measurement is not a straightforward option. The use of an LCR bridge to determine capacitance will require the same modelling considerations and assumptions as used in the FRA model. To provide an accurate estimate for each of the respective capacitances, two dimensional finite element analysis (FEA) was used based on the work from Section 6.5^4 . The FEA models are given in Appendix D.

The comparative results for the sectional capacitance based on an 8 section model, i.e. n = 8, are given in Table 9.6. When considered in the context that the parameters have been determined directly from FRA, satisfactory estimates have been obtained for the capacitance parameters (though a significant disparity is observed in C_{SXi}). It is proposed

 $^{^{4}}$ The HV winding series capacitance values can not be readily determined using two dimensional FEA. This parameter estimate was determined using (6.49) as derived in Chapter 6.

Parameter	Symbol	Units	Estimate
Initial core perm.	μ_i	${\rm Hm^{-1}}$	798
Core conductivity	σ_C	MSm^{-1}	2.1
Lamination thick.	2b	mm	0.35
Stacking factor	k	-	0.95
HV cond. area	A_X	mm^2	18.8
LV cond. area	A_x	mm^2	444
HV turns per disc	N_{LX}	-	25
LV layers	N_{Lx}	-	6
HV leak. induct.	L_{LX}	mH	101
LV leak. induct.	L_{Lx}	μH	2.1
HV Leak. coeff.	$ au_{HV}$	-	0.5
LV Leak. coeff.	$ au_{LV}$	-	10^{-4}
LV Winding Cap.	C_{Sxi}	pF	0.27
HV-Tank Loss	R_{gXi}	ΜΩ	2
HV-LV Loss	R_{Xxi}	ΜΩ	20
LV-Core Loss	R _{gxi}	ΜΩ	0.7
HV Winding Loss	R _{SXi}	kΩ	63
LV Winding Loss	R_{Sxi}	kΩ	10

Table 9.7: Parameter estimates.

that the estimation errors associated with these capacitance values are due to small errors in the modelling of complex permeability. Due to the highly non-linear nature of complex permeability (refer to Figure 3.13), even subtle variations could have a significant influence on the magnetising inductance with frequency. Any errors in inductance will inversely affect the estimated value of a capacitor during the model fitting. It is proposed that the accuracy of the capacitance estimates could be improved by incorporating additional features into the modelling of the transformer's core, such as the influence of core joints.

The overall parameter estimate results of Tables 9.5 and 9.6 are satisfactory and provide support for the physically representative nature of the transformer model. For the sake of completeness, the remaining parameter estimates are shown in Table 9.7.

9.7 FRA Interpretation Example

It has been proposed that the modelling approach provides a suitable platform to facilitate advances in the interpretation of FRA. A suitable test of the veracity of this proposal is to demonstrate that changes in the frequency response due to a deformation in a transformer winding could be *simulated* via appropriate parameter value changes in the transformer model, as demonstrated in [61, 65]. A more robust evaluation of the modelling approach would be to test if the model parameters would correctly change to reflect an altered winding geometry induced by deformation. The purpose of this section is to demonstrate the latter test via a practical example on the 1.3MVA 11kV/433V Dyn1 distribution transformer.

9.7.1 Short Circuit Forces and their Potential Consequences

A short circuit can place tremendous forces upon transformer windings and mechanical structure [69]. Analysis of the leakage flux fields during a short circuit shows that the forces can be decomposed into radial and axial components [25]. The radial leakage flux is predominantly at the winding ends and produces an axial force on the windings. This can result in an axial displacement of the HV winding with respect to the LV winding [61]. The leakage flux which passes across the core window in the axial direction (as discussed in Section 5.4) will produce an outward radial force on the HV winding and an inward radial force on the LV winding [25]. The outward radial force is tensile in nature and can stretch the conductor or break a poor joint, which can lead to failure. However, the relatively high tensile strength of the conductor material means that failures in the outer winding due to tensile stress are unlikely [71]. The inward radial force places compressive stress on the LV winding. This compression can lead to winding deformation known as buckling and is a common mode of failure [61]. Buckling of the LV winding is the focus of this FRA interpretation example.

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Figure 9.13: Buckling modes associated with radial stress on a transformer's LV winding; (a) forced buckling, (b) free buckling.

9.7.2 Buckling Modes for the LV Winding

When the support structure of the LV winding has a greater stiffness than the LV winding conductors themselves, under compressive stress it is possible for the winding conductors to bend in between each of the spacers towards the core. This is known as *forced buckling* [105] and is shown in Figure 9.13(a). *Forced buckling* in an LV winding will lead to an increase in the average distance between the HV and LV windings. This will result in a reduction in the HV to LV winding capacitance. Conversely, since the average distance between the LV winding and the core decreases, there is a corresponding increase in the LV to core capacitance [61].

Another buckling mode known as *free buckling* can occur when the conductor has a higher stiffness than the winding support structure. Under these circumstances the winding can buckle both inwards and outwards around the circumference [105]. This buckling mode is shown in Figure 9.13(b).

9.7.3 The Emulation of an Outward Radial Buckle in the LV Winding

A testing limitation placed on this research is for all tests conducted on transformers to be non-destructive. As a result, in order to obtain the FRA data associated with different levels of winding deformation, we propose to utilise a method which could emulate "buckling" in the LV winding, but would be temporary in nature so that the transformer can be restored to its original condition at the end of the testing program. Emulating the buckling of the LV winding requires the ability to reduce the HV to LV winding capacitance and increase the LV to core capacitance. Both of these capacitances can be estimated from the coaxial cylinder capacitance relationship of (6.23). With reference to (6.23), the only parameter that can be altered without significant mechanical change is ϵ , the electrical permittivity of the dielectric medium. A change in ϵ can be achieved by

9.7. FRA INTERPRETATION EXAMPLE



Figure 9.14: The emulation of an outward radial buckle in the LV winding using a neoprene rubber insert between the HV and LV windings.

changing the dielectric material between the cylinders.

Since the transformer available for the test is air cooled, the most practical method available to emulate "buckling" in the LV winding was to insert neoprene rubber between the HV and LV windings. Since neoprene rubber has an electrical permittivity of 6.7 (compared to 1 for air)[4], the inserts will have the effect of increasing the HV to LV capacitance, and will therefore approximately emulate an outward radial buckle in the LV winding. Though a buckle in the LV winding would typically be inward, the objective of this set of tests is to determine if the model parameters correctly change to reflect an altered winding "geometry", and hence validate the physically representative nature of the model.

9.7.4 Transformer "Buckle" Tests

The 1.3MVA 11kV/433V Dyn1 distribution transformer was used for the "buckle" emulation modifications. The modifications involved the insertion of 6mm neoprene rubber strips in between the Phase A HV and LV windings. Each of the strips ran the full axial length of the winding. Four "buckle" tests were conducted in total. The first test was an unmodified baseline test which is referred to as 0% "buckle". For the second test the transformer was modified such that the neoprene inserts covered 8% of the LV winding's outer circumference. This coverage was increased to 16% for the third test and to 24% for the fourth. HV Winding End to End Open Circuit, LV Winding End to End Open Circuit and Capacitive Interwinding FRA tests were then conducted on each of the "buckle" test cases. A zoomed in view of the resulting HV Winding End to End Open Circuit and



Figure 9.15: Zoomed in view of AC and BA High Voltage End to End Open Circuit FRA tests. The tests are based on winding "buckling" of 0%, 8%, 16% and 24%.

Capacitive Interwinding FRA tests is given in Figures 9.15 and 9.16 respectively. In both figures, frequency response changes coinciding with the increasing degree of "buckle" in the transformer's A phase windings are observed.

9.7.5 Parameter Estimation for Radial "Buckling"

As discussed in Section 9.7.2, radial buckling results in changes to certain parameters. Given a baseline FRA for comparison, knowledge of these parameters and the direction of their expected change can be used to our advantage. The first step is to run the estimation algorithm on the baseline FRA to determine the baseline parameter values. Other than the parameters specific to the deformation type being investigated, the remaining parameter values can all be fixed after this first run. The parameters of interest are then tightly constrained around their baseline estimates and the estimation algorithm is run with the FRA data that is in question. This approach significantly limits the degrees of freedom associated with the model, making the estimation algorithm more sensitive to the subtle changes that are being investigated.

The majority of power transformers are oil filled where the oil plays the dual role of insulation and cooling medium [53]. However, in order to facilitate internal tank access for measurement and modification, we have used an air cooled test transformer. The disadvantage of using an air cooled transformer for our tests is its sensitivity to atmo-



Figure 9.16: Zoomed in view of Aa and Bb Capacitive Interwinding FRA tests. The tests are based on winding "buckling" of 0%, 8%, 16% and 24%.

spheric variations such as temperature and humidity, which can vary widely throughout the course of a day. The variation in humidity during the week of transformer testing is shown in Figure 9.17. One area that is particularly sensitive to changes in both humidity and temperature is the losses associated with the permittivity of the insulation material [16]. Variation in the complex permittivity is reflected in the transformer's frequency response. As a result, the estimated parameter values may be influenced by changes in atmospheric conditions. In order to detect the subtle changes expected in the model parameters as a result of the "buckle" tests, we need to adopt an approach which will minimise the influence of both temperature and humidity.

This independence can be achieved in the following manner. Rather than look at the absolute value of a parameter, we can use the relative difference between similar parameters on different phases, e.g. C_{Aai} - C_{Bbi} . We use this difference as an *independent parameter* for each of the "buckle" test cases. This differential approach removes the common mode effects due to changes in the atmospheric conditions since only the absolute value of the parameters is affected.

Table 9.8 lists the interwinding capacitance estimates of each phase for each of the test cases. The changes observed in the capacitance are not always consistent due to the humidity and temperature induced variations in the complex permittivity of the insulation, the modelling accuracy limitations, and the subtle nature of the parameter changes. It is for this reason that the proposed relative parameter difference approach was utilised, and



Figure 9.17: Humidity variation observed during a week of transformer testing [Sourced from the Australian Bureau of Meteorology].

% Circum. "buckling"	C_{Aai}	C_{Bbi}	C_{Cci}	$C_{Aai} - C_{Bbi}$	$C_{Aai} - C_{Cci}$	\triangle_{AV}
0% (Baseline)	77.9	74.1	73.1	+3.8	+4.8	0
8%	79.8	74.7	73.6	+5.1	+6.2	+1.35
16%	80.2	72.5	71.7	+7.7	+8.5	+3.80
24%	80.9	72.0	69.9	+8.9	+11.0	+5.65

Table 9.8: "Buckling" test interwinding capacitance estimates for each phase. All values are in pF.

% Circum. "buckling"	% Change (Model)	% Change (FEA)
0% (Baseline)	_	_
8%	+2%	+3%
16%	+5%	+6%
24%	+7%	+9%

Table 9.9: Model and FEA estimates for the change in the Phase A interwinding capacitance based on the emulation of outward radial "buckling" for 0% (Baseline), 8%, 16%and 24% of the winding circumference.

Table 9.8 includes the relative difference between the estimated interwinding capacitance of phase A and that of phases B and C. The last column of the table lists Δ_{AV} which represents the average change in the capacitance difference relative to the baseline values. Therefore Δ_{AV} is an *independent* value for the change in the interwinding capacitance of phase A due to "buckling". As per Section 9.7.3, for the "buckling" conditions being considered here we would expect the relative difference between the phase A interwinding capacitance and that of phases B and C to increase with increased "buckle". The value of Δ_{AV} for each test case shows this to hold true.

In order to confirm that the degree of change in the interwinding capacitance is correct, the change relative to the baseline value is compared to that predicted by FEA in Table 9.9 (the FEA models are given in Appendix D). The estimated percentage change in the value of the capacitance relative to the amount of buckling is very close to that predicted by FEA.

This example provides support for the physically representative nature of the modelling approach developed in this thesis. It also provides an insight into its potential as a tool for assisting in FRA interpretation.

9.8 Conclusion

The purpose of this chapter was to demonstrate, by application, the physically representative nature of the transformer model proposed in this thesis and demonstrate how it can be used to facilitate improved FRA interpretation. To achieve this goal, FRA tests were performed on a Dyn1 1.3MVA 11kV/433V distribution transformer. The FRA testing procedure resulted in nine unique frequency responses (three connection permutations for each of the three FRA test types). An estimation algorithm was then applied. This algorithm simultaneously estimated each of the model transfer functions for the corresponding FRA data using a common parameter set with defined constraints.

The estimation results were very good for each of the FRA tests conducted. To confirm the physically representative nature of the FRA models, several key parameters whose value could be accurately determined through internal inspection were compared against their estimated counterparts. These parameters were all within a reasonable tolerance of their actual values, providing support for the physically representative nature of the model.

To demonstrate how the modelling approach could be used to facilitate improved FRA interpretation, a transformer was modified in order to emulate winding deformation. The parameter estimation algorithm was able to use FRA data to correctly determine the subtle variation in capacitance which was indicative of the induced change in the winding's structural "geometry".

Chapter 10

Conclusion and Further Research

10.1 Conclusion

The windings and associated mechanical structure of a transformer can undergo high levels of mechanical stress during a fault condition. Such mechanical stress can induce winding deformation that could lead to transformer failure. Frequency response analysis is a diagnostic tool that can be used to look for subtle changes in a transformer's frequency response [40]. Since deformation of the windings will change its frequency response, FRA is an ideal tool that can be used to monitor the mechanical integrity of the windings.

Current practice is for trained personnel to look for variation in a transformer's frequency response relative to historical or comparable transformer data. Cigre's A2.26 working group published a report in 2008 [6] that highlighted the need for research into models based on geometric parameters to improve FRA interpretation. The intention of the work presented in this thesis is to follow the Cigre recommendation and advance current research into geometric parameter based transformer modelling for FRA interpretation.

The preliminary step was to review the most common designs used in modern power transformer construction. From this it was proposed that the research focus on double wound, three phase, three limb core form power transformers. This is the nominal arrangement used in the construction of small to medium sized power transformers, and provides a wide application base. However, the approach described in this thesis is flexible enough to be modified to suit most modern power transformer designs.

A topic of contention in the area of transformer modelling is the frequency above which the core's complex permeability can be neglected. Current research suggests 1MHz. Based on the presented results in this thesis, the complex permeability is still significant at 1MHz and does not approach unity until frequencies above 15MHz, even for the largest lamination thicknesses. Complex permeability must therefore be taken into account for the entire FRA spectrum. In addition, it was demonstrated that during an FRA test the relative permeability approaches the initial permeability. As a result, the FRA data can be considered to have a level of independence with respect to commercially available FRA test equipment.

Having obtained a definition for the complex permeability of the core, a detailed analysis of the transformer's self, mutual and leakage inductance relationships was conducted. Inductance equations were derived based on measurable geometric parameters including the core yoke and limb lengths, core cross sectional area, and number of winding turns.

A generic phase lumped parameter model was then developed for a power transformer. The model was based on composite inductive, capacitive and resistive elements. Each inductive element incorporated the complex frequency dependent self, mutual and leakage inductance relationships that exist within and between each winding section. The model included a capacitive element for capacitance between windings, windings and the core, windings and the tank, the high voltage windings of different phases and the capacitance across a winding section. The capacitive elements also included an associated dielectric loss term. The resistive element incorporated the DC, skin and proximity effects of each winding section. The generic nature of this model accommodates the substitution of model parameters that are phase dependent.

To facilitate interpretation of FRA data the transformer model needs to incorporate the FRA test type, the terminal permutations, as well as the vector group of the transformer under consideration. To achieve this in a flexible manner, a layered modelling approach was adopted. The first layer was three instances of the generic phase model. They were nominally allocated generic phases X, Y and Z. The generic phase referencing facilitated the modelling of the different terminal permutations associated with an FRA test sequence. The second layer was the transformer vector group. This layer specifies the interconnection of the generic phase models. The third and final layer of this modelling approach was the FRA test type. This layer adds to the model the FRA voltage source and output impedance, both of which are FRA test dependent with respect to their location within the model. A complete model was then constructed for the High Voltage End to End, Low Voltage End to End, and Capacitive Interwinding FRA tests. Since this work has primarily focused on distribution level transformers, each of these models was based on a Dyn connection.

Other researchers have discussed the issues of producing a model that is not physically feasible. To overcome this issue, relationships have been developed which provide an initial parameter estimate, and corresponding constraints, for a number of key parameters. The constraints around these parameters provide a high degree of confidence in the physically representative nature of the final result. Since the detailed internal design specifications of a transformer are regarded as the intellectual property of the manufacturer, it is rarely available to utilities and testing authorities. As a result, the parameter constraints developed use readily available data such as external transformer dimensions, routine test data, and nameplate details. To demonstrate the relative accuracy of this approach, parameter estimates based on these relationships were successfully applied to a range of small to medium sized transformers. To demonstrate the veracity of the modelling approach, a case study on a distribution transformer was performed. FRA data was acquired for each of the three connection permutations of three different FRA test types. This resulted in nine unique FRA data sets. Utilising the initial parameter estimates and constraints, a constrained nonlinear optimisation algorithm was simultaneously applied to all nine data sets in order to determine a cost minimum. The cost related the cumulative residual between the corresponding model and data frequency points. The resulting parameter solution set was then used to plot the model frequency responses against their respective FRA test. The correlation between the model and data sets was excellent. To confirm that the model was physically representative, several of the key parameters were compared against their measured counterparts. Taking into account the fact that these parameters were determined without knowledge of the internal dimensions of the transformer, the results were very good.

To demonstrate how the model could be used to assist in FRA interpretation, the distribution transformer was modified in order to emulate an outward radial "buckle" in the LV winding. This was achieved by inserting neoprene strips between the HV and LV windings for a percentage of the winding circumference. The change in dielectric material for a proportion of the winding altered the effective capacitance between the windings. This change in capacitance was reflected in the transformer FRA. Through application of the parameter estimation algorithm, subtle changes in the HV to LV capacitance was detected. The percentage change in capacitance was very close to FEA model prediction. This result highlights the physically representative nature of the model and also shows the potential of this approach as a tool to support FRA interpretation.

10.2 Further Research

There are a number of areas that are targeted for future research. These areas include improvements to the current models, as well as areas that either directly or indirectly could benefit through the application of the work presented in this thesis.

10.2.1 Model Improvements

The model fitting for the High Voltage Winding End to End and Capacitive Interwinding FRA tests were very good. However, the results for the Low Voltage Winding End to End test could be further improved. In order to improve the model for this test, additional research is required into the model elements and structure reflecting the low voltage side of the transformer winding.

10.2.2 Expand Model Portfolio

The research in this thesis focused attention on Dyn connected transformers undergoing High Voltage Winding End to End Open Circuit tests, Low Voltage Winding End to End Open Circuit tests and Capacitive Interwinding tests. In order to be able to apply the proposed modelling approach more broadly, a valuable area of future work would be to expand the modelling portfolio to include a complete set of models for all possible permutations of vector group and FRA test type.

10.2.3 Fault detection using FRA Interpretation

The simultaneous fitting of multiple FRA test types can lead to more accurate parameter estimates. With more accurate parameter estimates, a clearer understanding of the physical phenomenon driving the frequency change is obtained. Using the model portfolio discussed in the previous section, the author would like to apply this approach to various transformer connections whilst simulating the most common faults. It is hoped that this approach would lead to a broader understanding of the fault to frequency response relationship, and hence improve FRA interpretation.

10.2.4 Partial Discharge Location

In the conference proceedings [79, 80] I present work which uses a narrowband single phase transformer model for partial discharge (PD) location. As an area of future work, I propose using a similar approach with the more comprehensive models presented in this thesis to determine PD location within a three phase transformer. In addition, unlike the approach presented in these articles, the wide-band model parameters would be preconfigured using FRA test data to improve modelling accuracy.

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Appendix A

Inductive Disparity

A.1 Introduction

We investigate the low frequency response of three different FRA tests on two transformers with different vector group numbers in this appendix. The research examines how inductive disparity coupled with different connection and measurement topologies can produce a significantly different frequency response. In order to explain the frequency response differences, simplified versions of the three phase generic model presented in Chapter 7 are developed for the HV Winding End to End Open Circuit, LV Winding End to End Open Circuit and Capacitive Interwinding FRA tests. To assist with the model reduction, it will be assumed that a large turns ratio exists between the high and low voltage windings. Other fundamental assumptions will be justified in the following sections.

This appendix is structured in the following manner. Section A.2 highlights the importance of understanding the influence inductive disparity has on a frequency response. Section A.3 justifies the inductive disparity hypothesis. Section A.4 looks at the variation in inductance between the HV and LV windings for the two transformers under consideration. Section A.5 looks at the variation between the various self and mutual inductance relationships for the two test transformers. Section A.6 then proposes simple models to explain the low frequency responses for each of the three FRA test types. Concluding remarks are then given in Section A.7.

A.2 Background

The disparity in inductance between the phases of a three phase transformer is a well documented property [41, 71, 88, 115]. The effective magnetic path length observed by a winding is inversely proportional to the winding's inductance [89]. For a three phase core type transformer, this results in the inductance of phase B being moderately larger than that of phase A and phase C, which are approximately equal. Inductive disparity can create significant variation between the observed frequency responses of an FRA test



Figure A.1: Magnetic equivalent circuit of a three phase two winding core type transformer

in the low frequency regions.

To detect change, testing personnel compare the FRA waveforms against historical data, sister transformer data or between phases on the same transformer, the latter test being the most convenient. It is the practice of interphase comparison that makes understanding the influence of inductive disparity and its relationship with a transformer's vector group so critical.

A.3 Inductive Disparity

FRA testing currents will generate an alternating magnetic field which will subsequently induce eddy currents in the transformer core. As discussed in Chapter 3, at high frequencies this effect will attenuate the complex permeability of the core. As such, the inductive disparity will dominate at lower frequencies (<10kHz). With reference to Figure A.1, \mathcal{F} represents the respective magnetomotive force (mmf) for each of the windings, $\mathcal{R}_{\rm E}$ is the core limb reluctance, $\mathcal{R}_{\rm Y}$ is the core yoke reluctance and $\mathcal{R}_{\rm L}$ is the winding leakage flux reluctance. The linear dimensions of the core are l_E for the mean limb length and l_Y for the mean yoke length.

The reluctance of a magnetic core section of mean length l and cross sectional area A_{CS} , with an effective permeability μ is defined as [89]:-

$$\mathcal{R} = \frac{l}{\mu A_{CS}} , \qquad (A.1)$$

where $\mu = \mu_0 \mu_R$, $\mu_0 = 4\pi \ge 10^{-7} \text{H/m}$, $\mu_R = \text{Relative permeability of the core}$.

It is assumed that the core cross sectional area and permeability are uniform across all core sections. In addition, flux leakage is small relative to core flux at low frequency and will therefore be neglected. From (4.15), the reluctance observed by windings on the outside limbs is,

$$\mathcal{R}_{\mathrm{MA}} = \mathcal{R}_{\mathrm{MC}} = \left[\frac{1}{\mu A_{CS}}\right] \left[\frac{\left(l_E + l_Y\right)\left(3l_E + l_Y\right)}{2l_E + l_Y}\right] \quad , \tag{A.2}$$

and from (4.24), the reluctance observed by windings on the centre limb is,

$$\mathcal{R}_{MB} = \left[\frac{1}{\mu A_{CS}}\right] \left[\frac{3l_E + l_Y}{2}\right] \quad . \tag{A.3}$$

In practical transformer design, l_E and l_Y are similar hence,

$$\mathcal{R}_{MB} < \mathcal{R}_{MA} = \mathcal{R}_{MC} . \tag{A.4}$$

Inductance is given by,

$$L = \frac{N^2}{\mathcal{R}} , \qquad (A.5)$$

where L is winding inductance and N is the number of winding turns. Since N is the same for each equivalent winding, from (A.4) and (A.5), the inductance of phase B is greater than that of phases A and C, i.e.

$$L_B > L_A = L_C . (A.6)$$

As is observed in Section A.6, the inductive variation, or disparity, of (A.6) has a significant influence upon the low frequency response of a transformer.

A.4 HV to LV Inductance Ratio

With reference to Figure A.2 (and as discussed in Section 2.9.1), it is noted that for a Dyn connected transformer there is a direct coupling of the HV winding line voltage v_{XY} to that of the LV winding phase voltage v_{xn} . This results in a connection based $\sqrt{3}$ voltage gain [121],

$$\frac{N_X}{N_x} = \frac{\sqrt{3}v_{XY}}{v_{xy}} , \qquad (A.7)$$



Figure A.2: Dyn vector group connection.

where v_{XY} and v_{xy} represent line voltages for the HV and LV windings respectively, and N_X and N_x the corresponding number of turns per winding. Both the HV and LV windings share the same magnetic circuit, hence the circuit reluctance parameter, \mathcal{R} , is the same. Therefore, from (A.5), the winding inductance ratio between the HV and LV windings of a Dyn connected transformer can be found in terms of the turns ratio,

$$\frac{L_X}{L_x} = \frac{N_X^2}{N_x^2} \,. \tag{A.8}$$

Substituting (A.7) into (A.8) and noting that both the distribution transformers under test are 11kV-433V,

$$\frac{L_X}{L_x} = 3\left(\frac{v_{XY}}{v_{xy}}\right)^2$$
$$= 3\left(\frac{11000}{430}\right)^2$$
$$L_X = 1963L_x$$
$$\therefore \quad L_X >> L_x . \tag{A.9}$$

The relationship (A.9) is necessary for model reduction purposes in the latter sections of this appendix due to the fact that at low frequencies, the low voltage winding inductance can be considered negligible relative to that of the high voltage winding.

A.5 Self and Mutual Inductance Relationships

The mutual inductance between the low and high voltage windings of a particular phase, noting that the coefficient of coupling will approach unity at low frequencies, can be defined as,

$$M_{xX} \approx \sqrt{L_x L_X}$$
 (A.10)

From (A.9) we have

$$M_{xX} \approx \sqrt{1963L_xL_x} \approx 44L_x$$

 $M_{xX} >> L_x$. (A.11)

Therefore, the mutual inductance between the low and high voltage windings on a given phase is an order of magnitude greater than the self inductance of the low voltage winding.

The interphase mutual inductance can be determined from the mutual inductance definition [89],

$$M_{jk} = \frac{\Lambda_{jk}}{i_j} , \qquad (A.12)$$

where M_{jk} is the mutual inductance between windings j and k, Λ_{jk} is the flux linkage between windings j and k, and i_j is the current in winding j. With reference to the three limb core of Figure A.1, at low frequencies where $\mu_R >> 1$, it is observed that the flux generated by a winding on a given phase has a flux linkage on the winding of another phase relative to the magnetic path reluctance ratio. For example, the flux generated by a winding on phase A is split between the magnetic paths of phases B and C based on the ratio of their path reluctance. We also note that whilst the reluctance of phase B is less than that of phases A and C, as shown in (A.4), the difference is only modest and as such, the following generic inequality can be deduced from (A.12),

$$M_{xX} > M_{xY} . (A.13)$$

This shows that the mutual inductance between the low and high voltage windings on the same phase is always greater than the mutual inductance between the low and high voltage windings of different phases. Similarly the self inductance of a winding on a particular phase can be assumed to be larger than its mutual inductance with a similar winding of different phase,

$$L_x > M_{xy} . (A.14)$$

A.6 FRA Test Results

On a three phase transformer, FRA tests are conducted in sets of three such that all phase combinations are included in the testing sequence. Correlation between the generic



Figure A.3: 3 phase transformer Dyn vector group topologies: (a) Dyn1, (b) Dyn11



Figure A.4: Simplified generic phase model of a High Voltage Winding End to End Open Circuit FRA test of a Dyn connected transformer

X = FRA Input / Transformer HV terminal

Y = FRA Output / Transformer HV terminal

 $L_X, L_Y, L_Z =$ Self inductance of delta connected HV windings

 M_{XY}, M_{XZ}, M_{YZ} = Mutual inductance between HV windings

phase (X, Y or Z) of the model and the true phase for a particular test can be made via reference to the appropriate tables (Tables A.1, A.2 and A.3). In addition, to highlight the influence that the vector group will have on results, testing was conducted on two transformers with different vector group numbers, Dyn1 and Dyn11 (Refer Figure A.3).

A.6.1 High Voltage Winding End to End Open Circuit FRA Test for Dyn1 and Dyn11 Vector Group Transformers

The High Voltage Winding End to End Open Circuit FRA test records the frequency response between two of the three high voltage transformer terminals. The third high voltage terminal and the low voltage terminals are left open circuit. This test is repeated for all three high voltage terminal combinations. From a modelling perspective, with

Vector Group	FRA Test $(v_{IN}v_{OUT})$	X/x	Y/y	\mathbf{Z}/\mathbf{z}
	AC	A/a	C/c	B/b
Dyn1	BA	B/b	A/a	$\mathrm{C/c}$
	СВ	C/c	B/b	A/a
	AB	A/a	B/b	$\mathrm{C/c}$
Dyn11	BC	B/b	$\mathrm{C/c}$	A/a
	CA	C/c	A/a	B/b

Table A.1: HV winding End to End Open Circuit FRA test: Generic phase reference for Figure A.4

reference to (A.9) for the two transformers under consideration, the influence of the disconnected low voltage winding can be considered negligible. At low frequencies a model of a Dyn connected transformer can be reduced to a series combination of two windings in parallel with the third, as shown in Figure A.4. To consider all three phase combinations for this test, it is convenient to present the model in a generic phase form.

Analysing the simplified model and noting (A.6) and (A.14), it can be shown that the equivalent inductance between the input and output terminals is larger when the generic winding X represents phase B. The test terminal combination for X to represent the phase B high voltage winding, is dependent upon the vector group of the transformer and is highlighted in Table A.1. The influence of the larger equivalent inductance at low frequencies for tests between terminals B and A for a Dyn1 vector group, or terminals B and C for a Dyn11 vector group, is observed in the frequency responses of Figures A.5 and A.6. One observable effect of inductive disparity, for test BA in Figure A.5 or test BC in Figure A.6, is a smaller magnitude in the frequency range between 100Hz and 2kHz. This region where the roll off is 20dB/dec, is referred to as the inductive roll off. Another variation in the frequency response can be observed at the self resonant frequency which marks the transition between the inductive and capacitive regions of the frequency which marks the transition between the inductive results in a lower self resonant frequency which is clearly visible for test BA in Figure A.5. and test BC in Figure A.6.

A.6.2 Capacitive Interwinding FRA Tests for Dyn1 and Dyn11 Vector Group Transformers

This test measures the frequency response between a high and low voltage terminal on a particular phase. In this test the remaining high and low voltage terminals are left open circuit. Again the test is repeated for all three high to low voltage terminal combinations. A low frequency generic phase model for this test is shown in Figure A.7. From (A.9), the low voltage winding inductance is negligible at low frequency with respect to the high voltage winding and approximates a short circuit. Neglecting for the moment the



Figure A.5: HV Winding End to End Open Circuit FRA test using a HP89410A Vector Analyser on a 1.3MVA 11kV/433V Dyn1 transformer between high voltage terminals A to C, B to A and C to B.

inductive disparity, if the distributed high voltage winding inductance is assumed equal for all phases, then the total voltage across the phase X winding is equal to that of the phase Z winding,

$$v_{LX(k)} \approx v_{LZ(n-k+1)}$$
 where $k = 1$ to n . (A.15)

Therefore the current that flows into either end of winding Y is,

$$i_{Y1} \approx i_{Yn}$$
 . (A.16)

It then follows that the voltage across winding Y will tend to zero, i.e.

$$v_{LY} \longrightarrow 0$$
. (A.17)

From (A.17), the high voltage windings can be approximated to be a parallel combination of L_X and L_Z . When either of these two generic windings represent the B phase high voltage winding, the effective inductance is larger than the parallel combination of the outer limb windings (phases A and C). The larger effective inductance combined with the interwinding capacitance will result in a lower resonant frequency. This effect is observed in Figure A.8 for the high to low voltage terminal tests Aa and Bb of the Dyn1 vector group and in Figure A.9 for the high to low voltage terminal tests Bb and Cc on the



Figure A.6: HV Winding End to End Open Circuit FRA test using a HP89410A Vector Analyser on a 200kVA 11kV/433V Dyn11 transformer between high voltage terminals A to B, B to C and C to A.

Vector Group	FRA Test $(v_{IN}v_{OUT})$	X/x	Y/y	\mathbf{Z}/\mathbf{z}
	Aa	A/a	C/c	B/b
Dyn1	Bb	B/b	A/a	$\mathrm{C/c}$
	Cc	$\mathrm{C/c}$	B/b	A/a
	Aa	A/a	B/b	C/c
Dyn11	Bb	B/b	$\mathrm{C/c}$	A/a
	Cc	$\mathrm{C/c}$	A/a	B/b

Table A.2: Capacitive Interwinding FRA test: Generic phase reference for Figure A.7

Dyn11 vector group. These results are highlighted in Table A.2.

A.6.3 Low Voltage Winding End to End Open Circuit FRA Test on Dyn1 and Dyn11 Vector Group Transformers

This FRA test records the frequency response between each of the low voltage terminals and the neutral connection. During the test, the remaining two low voltage terminals and three high voltage terminals are left open circuit. The test is repeated for each phase. The FRA testing circuit is of low impedance and as such, at low frequencies, the low voltage



Figure A.7: Generic low frequency model representing a Capacitive Interwinding FRA test of a Dyn connected transformer X = FRA Input / Transformer HV terminal x = FRA Output / Transformer LV terminal n = Number of distributed winding sections $L_X, L_Y, L_Z =$ Distributed self inductance of HV windings $L_x, L_y, L_z =$ Distributed self inductance of LV windings $M_{XY}, M_{XZ}, M_{YZ} =$ Mutual inductance between HV windings $C_{Xx}, C_{Yy}, C_{Zz} =$ Distributed HV to LV winding capacitances $i_{Y1} =$ Current into winding node 1 of HV winding Y $i_{Y(n)} =$ Current into winding node n of HV winding Y

star connected windings which have their terminals unconnected, can be neglected from the model. The generic phase low frequency Dyn based transformer model is shown in Figure A.10.

With reference to (A.6), (A.11) and (A.13) it is noted that the equivalent low frequency inductance between the testing terminals is dominated by the windings on phase X, and the most significant contribution is made by the high voltage winding as per (A.11). Based on this result, the B phase to neutral test (Bn) on both the Dyn1 and Dyn11 vector groups will have a smaller magnitude in the inductive roll off region of the frequency response when compared to the An and Cn tests. The Bn test will also have a lower self resonant frequency. These results can be observed in Figures A.11 and A.12. Table A.3 presents the generic phase model phase reference for this test.



Figure A.8: Capacitive Interwinding FRA test using a HP89410A Vector Analyser on a 1.3MVA 11kV/433V Dyn1. Tests between terminals A to a, B to b and C to c.

Vector Group	FRA Test $(v_{IN}v_{OUT})$	X/x	Y/v	\mathbf{Z}/\mathbf{z}
1	(111 001)	/	/ 5	/
	an	A/a	C/c	B/b
Dyn1	bn	B/b	A/a	$\mathrm{C/c}$
	cn	$\mathrm{C/c}$	$\rm B/b$	A/a
	an	A/a	B/b	$\mathrm{C/c}$
Dyn11	bn	B/b	$\mathrm{C/c}$	A/a
	cn	C/c	A/a	B/b

 Table A.3: LV Winding End to End Open Circuit FRA test: Generic phase reference for

 Figure A.10

A.7 Conclusion

In this appendix we developed simple generic phase models for the physical interpretation of low frequency FRA results for distribution transformers with different vector groups. We have shown that the influence of inductive disparity on FRA is dependent upon the transformer connection and testing topology. More specifically, the FRA response differences between a Dyn1 and a Dyn11 connected transformer are explained, contributing to the research area of transformer frequency response interpretation.



Figure A.9: Capacitive Interwinding FRA test using a HP89410A Vector Analyser on a 200kVA 11kV/433V Dyn11 Tests between terminals A to a, B to b and C to c.





- x = FRA Input / Transformer LV terminal
- n = FRA Output / Neutral terminal

 $L_X, L_Y, L_Z =$ Self inductance of delta connected HV windings

 $L_x = \operatorname{Self}$ inductance of phase **x** - Star connected LV winding

 $M_{XY}, M_{XZ}, M_{YZ} =$ Mutual inductance between HV windings

 M_{xX}, M_{xY}, M_{xZ} = Mutual inductance between LV winding x and HV windings



Figure A.11: Low Voltage Winding End to End Open Circuit FRA test using a HP89410A Vector Analyser on a 1.3MVA 11kV/433V Dyn1 Transformer. Tests conducted between terminals a to n, b to n and c to n.



Figure A.12: Low Voltage Winding End to End Open Circuit FRA test using a Doble M5300 SFRA test set on a 200kVA 11kV/433V Dyn11 Transformer. Tests conducted between terminals a to n, b to n and c to n.

Appendix B

Eddy Current Losses in the Winding

In Section 3.4.1 we made the assumption that eddy current based losses in the winding are small relative to core losses and can therefore be neglected. This appendix justifies this assertion.

In Section 6.4 it was shown using the Dowell Method [42, 48] that the AC resistance due to skin effect can be estimated from,

$$R_{se} = \frac{R_{dc}\xi}{2} \left[\frac{\sinh \xi + \sin \xi}{\cosh \xi - \cos \xi} \right], \tag{B.1}$$

where

$$\xi = \frac{d\sqrt{\pi}}{2\delta}$$

 \boldsymbol{d} is the conductor diameter and

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

is the skin depth, f the frequency in Hz, and the permeability and conductivity of the conductor material is given by μ and σ respectively.

Similarly for proximity effect, the AC resistance for the mth layer is,

$$R_{pe} = \frac{R_{dc}\xi}{2} \left[(2m-1)^2 \frac{\sinh\xi - \sin\xi}{\cosh\xi + \cos\xi} \right].$$
(B.2)

An orthogonal relationship exists between skin and proximity effects [85]. As a result, the two effects can be decoupled and an estimate for the total eddy current losses can be determined through the addition of both effects (B.1) and (B.2), hence

$$R_{ac} = R_{se} + R_{pe} . (B.3)$$

With reference to (B.1), (B.2) and (B.3), the winding loss can be estimated for the combined influence of proximity and skin effect. These losses are calculated for the



Figure B.1: Estimated AC Resistance for the 200 turn single layer test bed winding. a) Total AC Resistance, b) Skin Effect Resistance, c) Proximity Effect Resistance.



Figure B.2: Comparison between the estimated model of the magnetic loss resistance to the calculated winding loss due to proximity and skin effect. a) Estimated model magnetic loss resistance, b) Estimated winding AC resistance.

winding utilized in the test bed of Figure 3.6 for frequencies up to 1MHz. The estimated results for the winding loss, Figure B.1, indicate only a modest increase in resistance with frequency. With reference to equation (B.2), it can be observed that proximity effect

losses are dependent on the winding layer being considered. Since the test bed winding was constructed as a single layer to limit low frequency resonant modes, an additional effect was the minimisation of proximity effect losses.

In order to compare the estimated winding losses and the losses due to the core, it is necessary to determine an estimate for the core losses. This is accomplished by extracting the real component of the inductor impedance from the model estimate in Figure 3.12. From (3.27),

$$R^{''} = w\mu_s^{''}L_0 ,$$

where R'' is the magnetic loss resistance. The magnetic loss resistance can then be compared to the winding loss resistance, Figure B.2. Above a few kHz, the magnetic loss resistance is orders of magnitude larger than the estimated winding contribution. The significant difference in magnitude of the loss components indicates that it is appropriate to consider the winding resistance negligible for this particular experimental configuration.
Appendix C

Scattering Parameters

The voltage (current) of a signal has both a time and a space component. At lower frequencies, or over small distances, we can neglect the spatial aspect. However at high frequencies or over long distances, the spacial voltage (current) distribution must be considered [74]. This is often referred to as transmission line effects. Essentially the voltage and current can be considered as waves which travel in both directions [1].

Scattering parameters, or S parameters as they are commonly referred, are a useful method for characterising a transmission line network [1]. With reference to Figure C.1, the incident wave variables a_1 and a_2 are defined as,

$$a_1 = \frac{e_{i1}}{\sqrt{Z_0}} , \qquad (C.1)$$

$$a_2 = \frac{e_{i2}}{\sqrt{Z_0}} , \qquad (C.2)$$

where e_{i1} and e_{i2} are the incident voltage waves on the two port network with respect to the source and the load respectively. Z_0 is the transmission line characteristic impedance. Similarly, the reflection wave variables b_1 and b_2 are defined as,



Figure C.1: Two port network connected by transmission lines.

$$b_1 = \frac{e_{r1}}{\sqrt{Z_0}} ,$$
 (C.3)

$$b_2 = \frac{e_{r2}}{\sqrt{Z_0}} ,$$
 (C.4)

where e_{r1} and e_{r2} are the reflection voltage waves on the two port network with respect to the source and the termination respectively. The S parameter matrix of a two port is given by,

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$
(C.5)

where,

$$S_{11} = \frac{b_1}{a_1}|_{a_2=0} , \qquad (C.6)$$

$$S_{21} = \frac{b_2}{a_1}|_{a_2=0} , \qquad (C.7)$$

$$S_{12} = \frac{b_1}{a_2}|_{a_1=0} , \qquad (C.8)$$

$$S_{22} = \frac{b_2}{a_2}|_{a_1=0} . (C.9)$$

 S_{11} and S_{21} are determined by terminating the output port with the characteristic impedance. This has the same effect as setting a_2 to be zero since the matched load will absorb all of the incident wave. Similarly, S_{12} and S_{22} can be determined by terminating the input port with the characteristic impedance. This will have the same effect as setting a_1 to be zero.

As proposed in Chapter 3, the use of a network analyser facilitates a testing platform that is capable of taking into account transmission line effects associated with the test equipment. The network analyser was used in Chapter 3 to determine the parameter S_{21} which represents the forward voltage gain of the two port network.

Appendix D

FEA Models

This appendix reproduces the two dimensional finite element analysis models which were used to benchmark the accuracy of the transformer model estimates for a number of capacitance values in Chapter 9. The models are based upon the analytical calculations of Section 6.5 and internal measurements taken directly from the transformer under consideration. For example, for the HV to LV winding capacitance, a simple FEA model is created which consists of two cylinders. The outside cylinder has a diameter equal to the inside diameter of the HV winding and the inside cylinder has a diameter equal to the outside diameter of the LV winding. Similar methods are used for the LV winding to core capacitance, the interphase HV winding capacitance, and the HV winding to tank capacitance. In each of these cases the region between the electrodes is assumed to be air. For the winding "buckling" models, the HV to LV winding capacitance model was recreated with an adjustable region between the two cylinders representing the neoprene rubber insert and its material permittivity.



Figure D.1: FEA model for the high voltage to low voltage winding capacitance of the 1.3MVA 11kV/433V Dyn1 transformer.



Figure D.2: FEA model for the low voltage winding to core capacitance of the 1.3MVA $11 \rm kV/433V$ Dyn1 transformer.



Figure D.3: FEA model for the interphase high voltage winding capacitance of the 1.3MVA 11kV/433V Dyn1 transformer.



Figure D.4: FEA model for the high voltage winding to tank capacitance of the 1.3MVA 11kV/433V Dyn1 transformer; (a) Outside limb, (b) Centre limb.



Figure D.5: FEA model for the high voltage to low voltage winding capacitance of the 1.3MVA 11kV/433V Dyn1 transformer with 15% of the circumference influenced by "deformation".



Figure D.6: FEA model for the high voltage to low voltage winding capacitance of the 1.3MVA 11kV/433V Dyn1 transformer with 25% of the circumference influenced by "deformation".

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